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Strain history effect in cyclic  
plasticity

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## Abstract

The simple plastic shear, plastic torsion, cyclic plastic shear, cyclic plastic tension - torsion and out of phase tests of 18G2A steel were performed. The specimens were loaded monotonically and cyclically with constant effective strain rate under controlled amplitude of plastic strains at room temperature. Using the technique of successive unloadings (presented in (12)), the standard physical quantities and the stress jumps corresponding to opposite directions of plastic straining were measured. For the Huber - Mises yield surface such results enable to follow the evolution of kinematic and isotropic hardening for such programs. The obtained results shows new effects concerning the influence of strain history on cyclic plasticity. They are similar to that obtained for 21CrMoV57 steel (12) and can be used for verification of existing theoretical approaches.

## Zusammenfassung

In dieser Arbeit werden die einfache plastische Scherung, plastische Torsion, zyklische plastische Scherung, zyklische Zug-Torsion und 'out of phase'-Experimente an einem 18G2A-Stahl untersucht. Die Proben wurden monoton bzw. zyklisch mit konstanter effektiver Verzerrungsgeschwindigkeit unter kontrollierter Dehnungsamplitude bei Raumtemperatur belastet.

Mittels der Methode der sukzessiven Entlastungen (vgl. [12]) wurden die Spannungssprünge bei entgegengesetzten plastischen Verzerrungen gemessen.

Diese Ergebnisse ermöglichen für eine Huber-Mises-Fließfläche die Ermittlung der Evolution von isotroper und kinematischer Verfestigung.

Die erhaltenen Ergebnisse zeigen neue Effekte bezüglich des Einflusses der Verzerrungsgeschichte bei zyklischer Plastizität. Sie sind ähnlich den in [12] für den Stahl 21CrMoV57 ermittelten Resultaten und können für die Verifikation bestehender theoretischer Ansätze genutzt werden.

<u>Contents</u>	Page
1. Introduction	3
2. Experimental technique	4
3. Experimental results	7
4. Conclusions	13
References	14
Figure Captions	16
Figures	21

## 1. Introduction

The behaviour of solids under monotonic and cyclic plastic loading is very complex and even more complex when the strain history effects are taken into account. Since an anticipation of such behaviour for metals plays an important practical role there were proposed a lot of theories to describe it. Some of them (1-4) are based upon the assumption that, apart from traditional yield surface in the stress space, there exist one or more surfaces in a space of the tensors describing the motion of the centre of the yield surface. The evolution laws for the kinematic and the isotropic part of hardening are usually postulated first and then they are jointly verified by means of the data obtained from the measurements of global strain - stress curves. Such data are insufficient for unique separation of, mentioned above, two hardening components. They have been performed also the experimental investigations concerning the possible shape of yield surfaces and its properties under simple loading programs and under cyclic loading (5-11). The position and the shape of the yield surface were determined only at some chosen instants of a cyclic loading programs. The obtained results are still insufficient for satisfactory verification of the law of kinematic and isotropic hardening. So, for proper formulation of such concepts, there are required new experimental data giving the measurements of some additional quantity not only in some chosen points of strain history, but during all cyclic loading program, also inside single cycle. Such results are shown in (12) for 21CrMoV57 steel under tension - compression and pure torsion cyclic loading at room temperature. They were obtained using the new technique of successive unloadings.

The objective of this work is to verify those results for different material (18G2A steel) and to provide the new experimental data for another strain histories.

## 2. Experimental technique.

The evolution of yield surface center and the evolution of yield surface radius, during monotonic and plastic loading, were determined using the technique of successive unloadings. The detail description of this procedure is given in [12] and now the only main idea will be presented.

Let us denote by the deviatoric part of stress tensor  $\sigma_{ij}$ ,  $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$ . Most of the theoretical models in cyclic plasticity employ the notion of a yield function  $f$ , which can be written in the following form:

$$f(S_{ij} - \alpha_{ij}, H) = 0, \quad f(0, H) \leq 0 \quad (1)$$

where  $\alpha_{ij}$  ( $\alpha_{kk}=0$ ) is sometimes identified with a macroscopic measure of microstresses. The letter  $H$  denotes symbolically the set of other possible parameters which describe the history of plastic strains.

Let us discuss the basic program O-C of proportional plastic straining shown in Fig.1a. The program is interrupted at point A where the specimen is unloaded and reloaded into reverse direction until the conventional small value  $\eta$  (for instance  $\eta = 0.000005$ ) of the increment of  $e^P$  is achieved. Then, the former direction of plastic strain is applied. In such a way one can, in an experimental program, identify the stress vector  $S_{ij}^R$  (Fig.1b). Its one end lies at the yield surface at point B' where plastic strain - rate vector has the direction opposite to that prescribed in the basic program. The value of  $S_{ij}^R$ , in chosen moment of the strain history, is those additional physical quantity, mentioned in introduction, useful for satisfactory verification of the kinematic and isotropic hardening at point A.

Let us denote:

$$Y_{ij} = (S_{ij} - S_{ij}^R)/2, \quad \Pi_{ij} = (S_{ij} + S_{ij}^R)/2 \quad (2)$$



It is possible to show, that for yield surface approximated by the Huber - Mises sphere (the length of radius  $R$  is independent on  $n_{ij}$ )

$$Y_{ij} = n_{ij} R(H), \quad \Pi_{ij} = \alpha_{ij} \quad (2a)$$

where  $n_{ij}$  - unit "vector" of directions in  $S_{ij}$  space.

Considering the projection of  $\Pi_{ij}$  and  $Y_{ij}$  onto the plastic straining direction ( $m_{ij}$  - the unit "vector" describing the plastic straining direction,  $n_{ij} = m_{ij} (\text{sgn } e^P)$ ):

$$Y = |Y_{ij} m_{ij}| = |(S_{ij} - S_{ij}^R) m_{ij}| / 2 \geq 0 \quad (3)$$

$$\Pi = m_{ij} m_{ij} = (S_{ij} + S_{ij}^R) m_{ij} / 2$$

The interpretation of  $Y_{ij}$  and  $\Pi_{ij}$  follows from (2a):

$$Y = |m_{ij} n_{ij}| R, \quad \Pi = \alpha_{ij} m_{ij} \quad (3a)$$

Thus  $\Pi$  is the projection of  $\alpha_{ij}$  on the direction  $m_{ij}$ , whereas  $Y$  equals the principal radius of an ellipsoide whenever  $|m_{ij} n_{ij}| = 1$ .

It means, that having  $S_{ij}$  and  $S_{ij}^R$  values in proportional loading proces, and assuming the Huber - Mises yield criterion, it is possible to define the yield surface center and the yield surface radius at chosen points of the plastic strain history. Because of simplicity of this technique, many points can be investigate (also, for instance, inside single cycle) and the evolution laws for kinematic and isotropic hardening can be experimentally defined.

In Fig.2 it is shown the application of the unloading technique for monotonic torsion. The specimen was loaded by torsion with the constant effective strain rate  $e_e^P = 3.4 \times 10^{-4} / s$ . After every effective plastic strain increment  $\Delta e_e^P = 0.008$  the stress tensor value  $S_{ij}$  was taken into memory, and the torque changes its direction to opposite. The slope of the unloading curve was measured. At point B (due to rheological effects) it was equal to the Young modulus for the virgin material and straight line "a" of the same slope was determined. The distance between the successive points at the unloading curve (or loading in "opposite" direction) and this line was then calculated. When it was equal to the

"offset definition" the another yield point  $S_{ij}^R$  was found and the torque changed its direction to the initial one. Having  $S_{ij}$  and  $S_{ij}^R$ , in the chosen points of strain history, the yield surface radius ( $Y$ ) and the yield surface center ( $\Pi$ ) was determined according to (3). Similar way such a values were found also for cycling loading even inside the single cycle.

The objective of this paper is the experimental determination of  $Y_1$  and  $\Pi_1$ , during cyclic and monotonic loading of a specimen under controlled plastic strain. The experiments concern three cases:

a) simple plastic shear,

$$\begin{aligned}\Pi_1 &= \sqrt{(3/2)}\Pi = \sqrt{3} (\tau + \tau^R)/2 \\ Y_1 &= \sqrt{(3/2)}Y = \sqrt{3} |(\tau - \tau^R)|/2\end{aligned}\quad (4)$$

b) simple plastic tension (compression),

$$\begin{aligned}\Pi_1 &= \sqrt{(3/2)}\Pi = (\sigma + \sigma^R)/2 \\ Y_1 &= \sqrt{(3/2)}Y = |(\sigma - \sigma^R)|/2\end{aligned}\quad (5)$$

where  $\Pi_1 = \sqrt{(3/2)}\Pi$  and  $Y_1 = \sqrt{(3/2)}Y$ .

c) out of phase plastic loading.

The experimental programs were performed at room temperature on thin tubes (outer diameter 24 mm., wall thickness 2 mm., gauge length 38 mm.) made of 18G2A steel. Tension - compression and torsion in opposite directions of cyclic programs were made using the Schenck tension - compression - torsion machine connected "on line" with the HP 1000 computer (machine and computer sponsored by Stiftung Volkswagenwerk). Force and moment acting on the specimen, elongation and torsion of the specimen, and other possible 35 measurements were read by a computer, then elaborated and the results obtained were used for continuous machine control. In this case it was symmetric cyclic program for fixed plastic strain amplitude with a set of unloadings (loading in opposite direction in order to determine the opposite point of the yield surface). All programs were performed at a constant effective strain rate  $e_e^P = 3.4 \cdot 10^{-4}/s$ , and the actual stresses versus the logarithmic plastic strain coordinates were calculated and plotted

( $\epsilon = \ln(l/l_0)$  ,  $\sigma = P/F_a$  , where  $l_0$  - gauge length,  $F_a$  - current specimen cross - section,  $P$  - force). Yielding was determined by the offset definition of  $\epsilon = 0.0005$ , what was discussed before.

### 3. Experimental results

#### Experiment 1. Monotonic torsion with unloadings.

The specimen was loaded by torsion at the constant effective strain rate  $\dot{\epsilon}_e^P = 3.4 \times 10^{-4} / s$  . After every effective plastic strain increment  $\Delta \epsilon_e^P = 0.008$  , the unloading technique was used to determine the position of yield surface center (assuming the Huber - Mises yield criterion) and the yield surface radius. This way it was possible to establish the loading path for both  $\Pi_1$  and  $Y_1$  (Fig.3).

#### Experiment 2. Monotonic tension with unloadings.

The specimen was loaded by tension with this same effective strain rate as in Experiment 1. Using the similar experimental technique the loading path for  $\Pi_1$  and  $Y_1$  was established (Fig.4).

#### Experiment 3. Cyclic torsion with following plastic strain

$$\text{amplitudes } \gamma^P / \sqrt{3} = \begin{matrix} +0.005, +0.010, +0.015, +0.020 \\ +0.030, -0.005 \end{matrix}$$

The specimen was cyclically loaded with gradually increasing plastic strain amplitude  $\gamma^P / \sqrt{3} = +0.005 - +0.030$  and than the amplitude  $\gamma^P / \sqrt{3} = -0.005$  was applied. Every strain amplitude change took place after reaching the steady state cycle for the former amplitude value. Using the unloading technique, the yield surface center movement and the evolution of yield surface radius for the first cycle and for the steady loop at every strain amplitude was

defined.

In Fig.5 are shown the stress-plastic strain curves (thick lines) and the  $\Pi_1$  curves (thin lines) for increasing strain amplitudes  $\gamma^P/\sqrt{3} = \pm 0.005 - \pm 0.030$ . The solid lines are drawn for the first cycle for certain amplitude and the dashed one for the stabilized loop.

In Fig.6 there are compared the stress - plastic strain curves, the  $\Pi_1$  curves and the  $Y_1$  curves for strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$  for virgin material (Fig.6a) and for material with history of growing plastic strain amplitudes  $\gamma^P/\sqrt{3} = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.030$  (Fig.6b).

In Fig.7 the skeletal points (points of maximum value of  $\sqrt{3}\tau$ ,  $|\Pi_1|$ , and  $Y_1$  for chosen strain amplitude at steady loop) for the stress - plastic strain curves, the  $\Pi_1$  curves and the  $Y_1$  curves (for mentioned cyclic program) are shown. They are compared with curves for monotonic load (Experiment 1  $-\sqrt{3}\tau$ ,  $\Pi_1, Y_1$ ) in Fig.8.

Experiment 4. Cyclic tension - compression with following strain amplitudes  $\varepsilon_x^P = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020$

Similar cyclic program as in Experiment 3 was repeated for tension - compression. The specimen was cyclically loaded with gradually increasing plastic strain amplitude  $\varepsilon_x^P = \pm 0.005 - \pm 0.020$  and than the amplitude  $\varepsilon_x^P = \pm 0.005$  was applied. Every strain amplitude change took place after reaching the steady state cycle for the former amplitude value.

In Fig.9 are shown the stress-plastic strain curves (thick lines) and the  $\Pi_1$  curves (thin lines) for increasing strain amplitudes  $\varepsilon_x^P = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020$ . The solid line denotes the first cycle for certain amplitude and the dashed one - the stabilized loop.

In Fig.10 there are compared the stress - plastic strain curves, the  $\Pi_1$  curves and the  $Y_1$  curves for strain amplitude  $\varepsilon_x^P = \pm 0.005$  for virgin material (Fig.10a) and for material with history of growing plastic strain amplitudes  $\varepsilon_x^P = \pm 0.005, \pm 0.010$

$\pm 0.015$ ,  $\pm 0.020$  (Fig.10b).

In Fig.11 the skeletal points (points of maximum value of  $|\sqrt{3}\tau|$ ,  $|\Pi_1|$ , and  $Y_1$  for chosen strain amplitude at steady loop) for the stress - plastic strain curves, the  $\Pi_1$  curves and the  $Y_1$  curves (for mentioned cyclic program) are shown. They are compared with curves for monotonic load (Experiment 2  $-\sqrt{3}\tau$ ,  $\Pi_1$ ,  $Y_1$ ) in Fig.12.

#### Experiment 5. Plastic prestrain in torsion followed by cyclic torsion loading

The specimen was plastically prestrained in torsion up to  $\gamma^P/\sqrt{3}=0.092$  and then cyclically loaded with plastic strain amplitude  $\gamma^P/\sqrt{3}=\pm 0.005$ . The stress - strain (thick line),  $\Pi_1$  (thin line), and  $Y_1$  cyclic curves are shown in Fig.13. Solid lines show the values during the first cycle and the dashed lines for stabilized loops.

#### Experiment 6. Plastic prestrain in tension followed by cyclic tension - compression loading

The specimen was plastically prestrained in tension up to  $\epsilon_x^P=0.092$  and then cyclically loaded with plastic strain amplitude  $\epsilon_x^P=\pm 0.005$ . The stress - strain (thick line),  $\Pi_1$  (thin line), and  $Y_1$  cyclic curves are shown in Fig.14. Solid lines show the values during the first cycle and the dashed lines for stabilized loops.

#### Experiment 7. Influence of plastic prestrain on the subsequent cyclic behaviour.

The specimen was first cyclically loaded in torsion with plastic strain amplitude  $\gamma^P/\sqrt{3}=\pm 0.005$  up to steady loop. Then it was plastically prestrained up to  $\gamma^P/\sqrt{3}=0.092$  followed by cyclic loading with plastic strain amplitude  $\gamma^P/\sqrt{3}=\pm 0.005$  up to steady state. The stress - strain,  $\Pi_1$  and  $Y_1$  curves for virgin material

(Fig.15a) and material after plastic prestrain (Fig.15b) are shown in Fig.15. Solid lines show the values for the first cycle and the dashed lines - for the stabilized loops.

#### Experiment 8. Tension compression and torsion alternating cyclic program

The specimen was cyclically loaded in tension - compression with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.015$  up to stabilized loop. Then, the cyclic loading in torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.015$  was applied, also up to steady state. This program was followed by cyclic plastic loading with following amplitudes:

$$\begin{aligned} \gamma^P/\sqrt{3} &= \pm 0.005 && \text{in torsion,} \\ \epsilon_x^P &= \pm 0.005 && \text{in tension - compression and} \\ \gamma_x^P/\sqrt{3} &= \pm 0.005 && \text{in torsion.} \end{aligned}$$

The stress - strain,  $\Pi_1$  and  $\gamma_1$  curves for this program are shown in Fig.16a and Fig.16b. As before the solid lines show the values for the first cycle and the dashed lines - for the stabilized loops.

#### Experiment 9. Out of phase loading with gradually increasing effective strain values: 0.005, 0.010 and 0.015

The specimen was loaded in out of phase manner shown in Fig.17. At the beginning the axial tension (or pure torsion) was applied up to chosen effective strain value (for instance  $\epsilon_x^P = 0.005$ ). Then the ratio  $\sqrt{3}\epsilon_x^P/\gamma^P$  was changing but the effective strain value  $\epsilon_e = (\epsilon_x^P + \gamma^2/3)^{0.5}$  was kept constant. Such a loading, which in this coordinates is represented by the circle (one circle - one cycle), was repeated up to steady state and then, the new effective strain value was applied. In Fig.18 the uniaxial tension stress values and pure torsion stress values are shown for following out of phase cycles.

Skeleton points for tension (maximal uniaxial tension stress in steady loop) and torsion (maximal torsion stress  $\sqrt{3}\tau_{\max}$  in steady

loop) for constant effective strain values: 0.005, 0.010 and 0.015 are shown in Fig.19. They are compared with monotonic load for tension (Fig.19a) and torsion (Fig.19b) and with skeleton points for cyclic tension - compression plastic loading and cyclic torsion plastic loading.

Experiment 10. Out of phase loading for following effective strain values: 0.005, 0.010 and 0.005

The specimen was loaded in out of phase manner with constant effective strains 0.005 up to steady state. Then, the new value of effective strains ( $\epsilon_e^P=0.010$ ) was applied up to new steady loop and so on. The uniaxial stress values and pure torsion stress values for following cycles of this history are shown in Fig.20.

Experiment 11. Alternating program of cyclic plastic loading: torsion, tension - compression and out of phase

The specimen was cyclically loaded in torsion with plastic strain amplitude  $\gamma^P/\sqrt{3}=\pm 0.005$  up to steady state. Then the load was changed to cyclic tension - compression with strain amplitude  $\epsilon_x^P=\pm 0.005$  followed by:

out of phase cycling for  $\epsilon_e^P=0.005$

out of phase cycling for  $\epsilon_e^P=0.010$

out of phase cycling for  $\epsilon_e^P=0.005$

Then the tension - compression cyclic loading with plastic strain amplitude  $\epsilon_x^P=\pm 0.005$  was applied followed by torsion cyclic loading with strain amplitude  $\gamma^P/\sqrt{3}=\pm 0.005$ . Every strain amplitude change took place after reaching the steady state cycle for former amplitude value. The stress - strain,  $\Pi_1$  and  $Y_1$  curves for tension - compression cyclic loading with plastic strain amplitude  $\epsilon_x^P=\pm 0.005$  just before out of phase loading program and after this program are shown in Fig.21. Solid lines show the values for the first cycle for chosen amplitude, and dashed lines - for stabilized loops.

**Experiment 12. Alternating program of cyclic plastic loading:  
tension - compression, torsion and out of phase**

The specimen was cyclically loaded in tension - compression with plastic strain amplitude  $\varepsilon_x^P = \pm 0.005$  up to steady state. Then the load was changed to cyclic torsion with strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$  followed by out of phase cycling for  $\varepsilon_e^P = 0.010$ . Then, the torsion cyclic loading with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$  was applied followed by tension - compression cyclic loading with strain amplitude  $\varepsilon_x^P = \pm 0.005$ . Every strain amplitude change took place after reaching the steady state cycle for former amplitude value. The stress - strain,  $\Pi_1$  (Fig.22a) and  $Y_1$  (Fig.22b) curves for tension - compression cyclic loading and torsion cyclic loading before and after out of phase loading program are shown in Fig.22. Solid lines show the values for the first cycle of chosen amplitude, and dashed lines for the stabilized loops.

**Experiment 13. Monotonic torsion load after history of torsion cyclic plasticity with following amplitudes:  
 $\gamma^P/\sqrt{3} = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.030, \pm 0.005$**

The specimen was cyclically loaded in torsion with mentioned above plastic strain amplitudes. Every amplitude change took place after reaching the steady state cycle for former amplitude value. Then, the monotonic torsion load was applied. The monotonic plastic strain curve,  $\Pi_1$  and  $Y_1$  curves are shown in Fig.23.



Experiment 14. Monotonic tension load after history of tension - compression cyclic plasticity with following amplitudes:  $\epsilon_x^P = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.005$

In this experiment the similar program to that in Experiment 13 was performed. The specimen was cyclically loaded in tension - compression with mentioned above plastic strain amplitudes. Then, the monotonic tension was applied. The monotonic plastic strain curve,  $\Pi_1$  and  $\gamma_1$  curves are shown in Fig.24.

#### 4. Conclusions

The simple plastic shear, plastic torsion, cyclic plastic shear, cyclic plastic tension - torsion and out of phase tests of 18G2A steel were performed. The specimens were loaded monotonically and cyclically with constant effective strain rate under controlled amplitude of plastic strains at room temperature. Using the technique of successive unloadings (presented in (12)), the standard physical quantities and the stress jumps corresponding to opposite directions of plastic straining were measured. For the Huber - Mises yield surface the obtained results enable to follow the evolution of kinematic and isotropic hardening for such programs. The obtained results shows new effects concerning the influence of strain history on cyclic plasticity. They are similar to that obtained for 21CrMoV57 steel (12) and can be used for verification of existing theoretical approaches.

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## FIGURE CAPTIONS

- Fig 1. Theoretical basis of unloading technique.
- Fig 2. Application of unloading technique for monotonic tension.
- Fig 3. Stress-strain,  $\Pi_1$  and  $Y_1$  curves for monotonic torsion.
- Fig 4. Stress-strain,  $\Pi_1$  and  $Y_1$  curves for monotonic tension.
- Fig 5. Stress-strain and  $\Pi_1$  - curves for cyclic torsion with increasing plastic strain amplitudes:  $\gamma^P/\sqrt{3} = \pm 0.005$ ,  $\pm 0.010$ ,  $\pm 0.015$ ,  $\pm 0.020$ ,  $\pm 0.030$ .  
(solid lines - the first cycles, dashed lines - the stabilised cycles)
- Fig 6. Stress-strain,  $\Pi_1$  and  $Y_1$  curves for cyclic torsion with plastic strain amplitude  
a) for virgin material  
b) for material with history of increasing plastic strain amplitudes:  $\gamma^P/\sqrt{3} = \pm 0.005 - \pm 0.030$   
(solid lines - the first cycles, dashed lines - the stabilised cycles)
- Fig 7. Skeleton curves for stress ( $\sqrt{3}\tau_s$ ),  $\Pi_1$  ( $\Pi_s$ ) and  $Y_1$  ( $Y_s$ ) values for cyclic torsion in the case of increasing plastic strain amplitudes ( $\gamma^P/\sqrt{3} = \pm 0.005 - \pm 0.030$ ) and skeleton points after decreasing the strain amplitude to  $\gamma^P/\sqrt{3} = \pm 0.005$ .

- Fig 8. Comparison between the stress-strain,  $\Pi$  and  $\gamma_1$  curves for monotonic torsion and the skeleton points for stress ( $\sqrt{3}\tau_s$ ),  $\Pi_1$  ( $\Pi_s$ ) and  $\gamma_1$  ( $\gamma_s$ ) for cyclic torsion in the case of increasing plastic strain amplitudes ( $\gamma^P/\sqrt{3} = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.030$ ).
- Fig 9. Stress-strain and  $\Pi_1$  - curves for cyclic tension-compression with increasing plastic strain amplitudes:  $\epsilon_x^P = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020$ . (solid lines - the first cycles, dashed lines - the stabilised cycles)
- Fig 10. Stress-strain,  $\Pi_1$  and  $\gamma_1$  curves for cyclic tension-compression with plastic strain amplitude  $\epsilon_x^P = \pm 0.005$   
 a) for virgin material  
 b) for material with history of increasing plastic strain amplitudes:  
 (solid lines - the first cycles, dashed lines - the stabilised cycles)
- Fig 11. Skeleton curves for stress ( $\sigma_s$ ),  $\Pi_1$  ( $\Pi_s$ ) and  $\gamma_1$  ( $\gamma_s$ ) values for cyclic tension-compression in the case of increasing plastic strain amplitudes ( $\epsilon_x^P = \pm 0.005 - \pm 0.020$ ) and skeleton points after decreasing the strain amplitude to  $\epsilon_x^P = \pm 0.005$
- Fig 12. Comparison between the stress-strain,  $\Pi_1$  and  $\gamma_1$  curves for monotonic tension and the skeleton points for stress ( $\sigma_s$ ),  $\Pi_1$  ( $\Pi_s$ ) and  $\gamma_1$  ( $\gamma_s$ ) for cyclic tension-compression in the case of increasing plastic strain amplitudes ( $\epsilon_x^P = \pm 0.005 - \pm 0.020$ )

Fig 13. Stress-strain,  $\Pi_1$  and  $Y_1$  cyclic torsion curves for plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$  after plastic torsion prestrain  $\gamma^P/\sqrt{3} = 0.092$ .  
(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 14. Stress-strain,  $\Pi_1$  and  $\gamma_1$  cyclic tension-compression curves for plastic strain amplitude  $\epsilon_x^P = \pm 0.005$  after plastic tension prestrain  $\epsilon_x^P = 0.092$ .  
(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 15. Stress-strain,  $\Pi_1$  and  $\gamma_1$  curves for cyclic torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$   
a) for virgin material  
b) for material after plastic torsion prestrain  $\gamma^P/\sqrt{3} = 0.092$   
(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 16. Stress-strain,  $\Pi_1$  (Fig.16a) and  $\gamma_1$  (Fig.16b) curves for following cyclic program:  
tension-compression with plastic strain amplitude  $\epsilon_x^P = \pm 0.015$   
reverse torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.015$   
reverse torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$   
tension-compression with plastic strain amplitude  $\epsilon_x^P = \pm 0.005$   
reverse torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$   
(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 17. The out of phase loading (one circle = one cycle)

Fig 18. Maximum stress values ( $\sigma_{\max}$ ,  $\sqrt{3}\tau_{\max}$ ) as a function of cycle number in the case of out of phase loading with gradually increasing effective strain values:  $\epsilon_e^p = 0.005$ , 0.010 and 0.015.

Fig 19. Comparison between the stress-strain curves for monotonic tension (Fig.19a) (torsion-Fig.19b) and the skeleton points for stress in the case of tension-compression cyclic loading (Fig.19a) (cyclic torsion-Fig.19b), and the out of phase skeleton points.

Fig 20. Maximum stress values ( $\sigma_{\max}$ ,  $\sqrt{3}\tau_{\max}$ ) as a function of cycle number in the case of out of phase loading for following effective strain values:  $\epsilon_e^p = 0.005, 0.010$  and 0.005.

Fig 21. Stress-strain,  $\Pi_1$  and  $Y_1$  curves for cyclic tension-compression with plastic strain amplitude

- a) for virgin material
- b) for material after out of phase cyclic loading with following strain amplitudes:  $\epsilon_e^p = 0.005, 0.010, 0.005$ .  
(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 22. Stress-strain,  $\Pi_1$  (Fig.22a) and  $\gamma_1$  (Fig.22b) cyclic curves

Fig.22a/1 -tension-compression with plastic strain amplitude

Fig.22b/1  $\epsilon_x^P = \pm 0.005$  followed by cyclic torsion with plastic strain amplitude  $\gamma^P/\sqrt{3} = \pm 0.005$

Fig.22a/2 -cyclic torsion with plastic strain amplitude

Fig.22b/2  $\gamma^P/\sqrt{3} = \pm 0.005$  followed by cyclic tension-compression with plastic strain amplitude  $\epsilon_x^P = \pm 0.005$  for material after out of phase cycling with constant effective strain  $\epsilon_e^P = 0.010$ .

(solid lines - the first cycles, dashed lines - the stabilised cycles)

Fig 23. Stress-strain,  $\Pi_1$  and  $\gamma_1$  curves for monotonic torsion after torsion cyclic program for following plastic strain amplitudes:  $\gamma^P/\sqrt{3} = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.030, \pm 0.005$ .

Fig 24. Stress-strain,  $\Pi_1$  and  $\gamma_1$  curves for monotonic tension after tension - compression cyclic program for following plastic strain amplitudes:  $\epsilon_x^P = \pm 0.005, \pm 0.010, \pm 0.015, \pm 0.020, \pm 0.005$ .



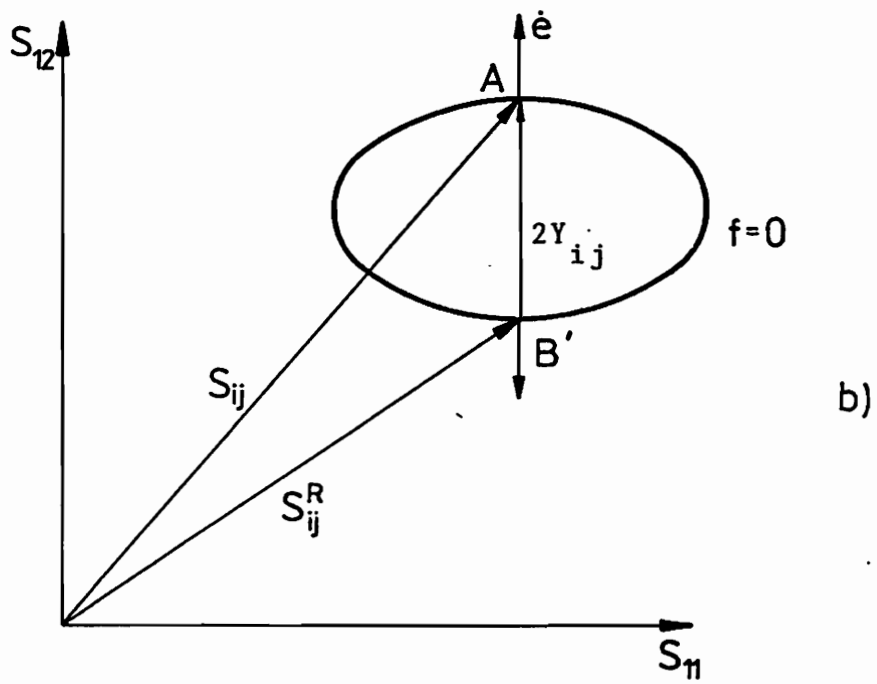
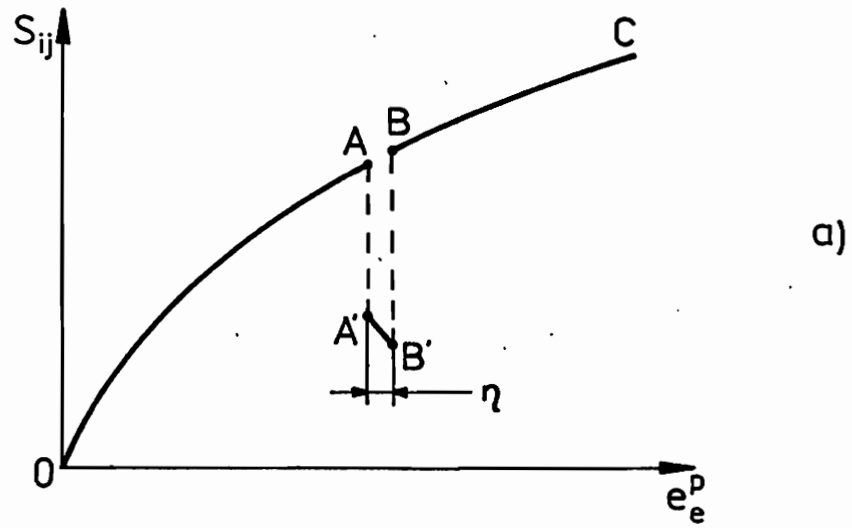


Fig.1

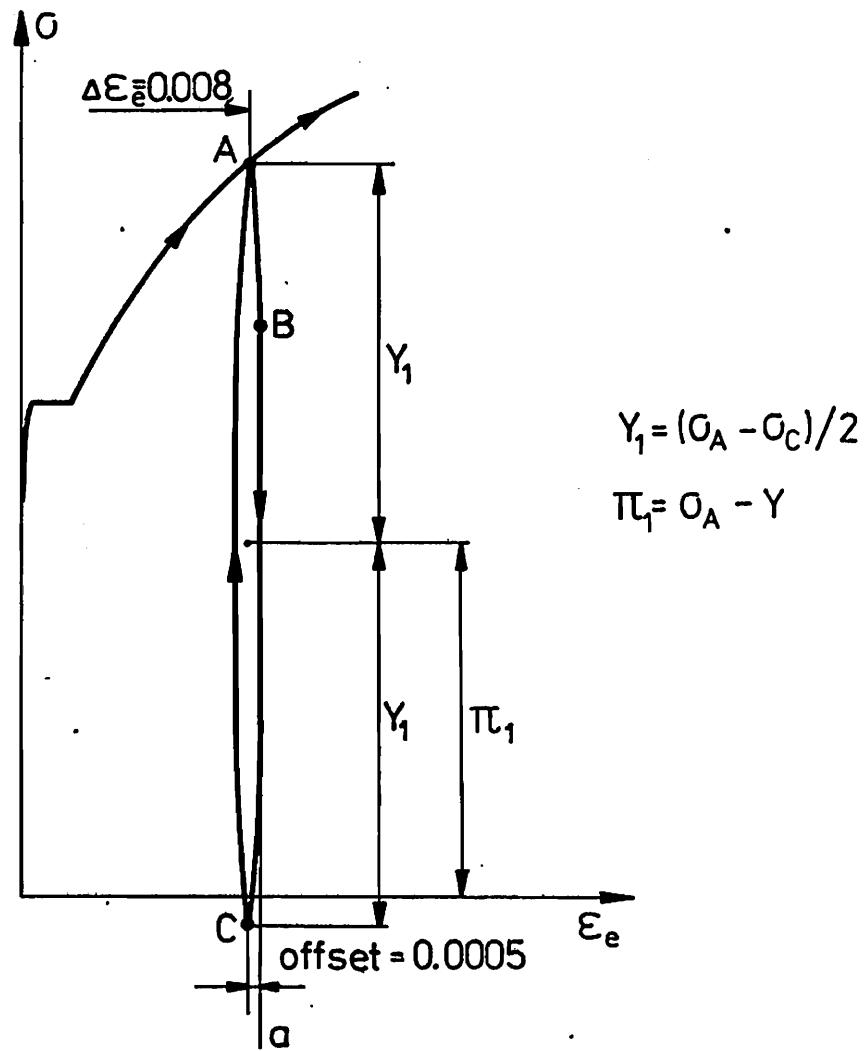


Fig.2

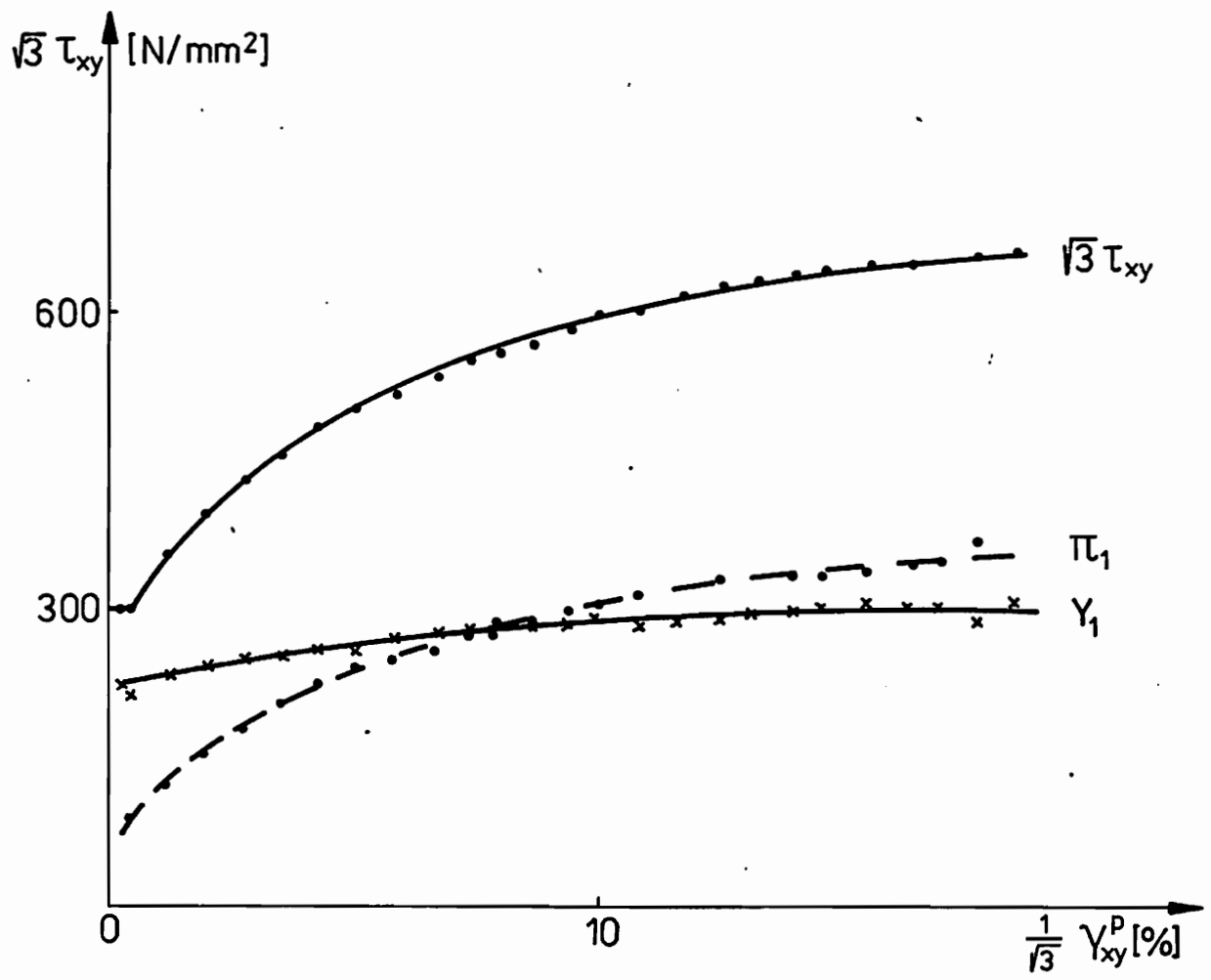


Fig. 3

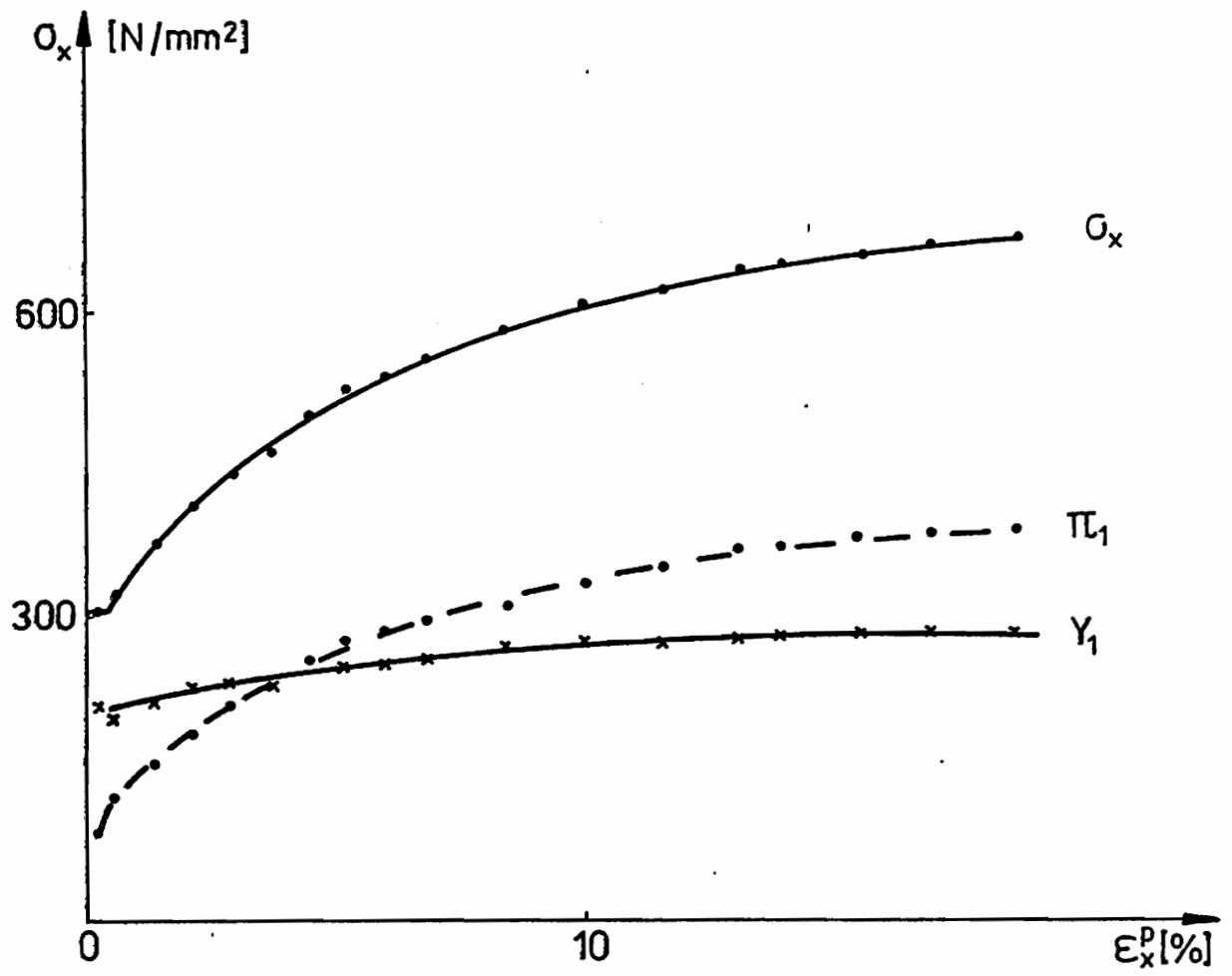


Fig.4

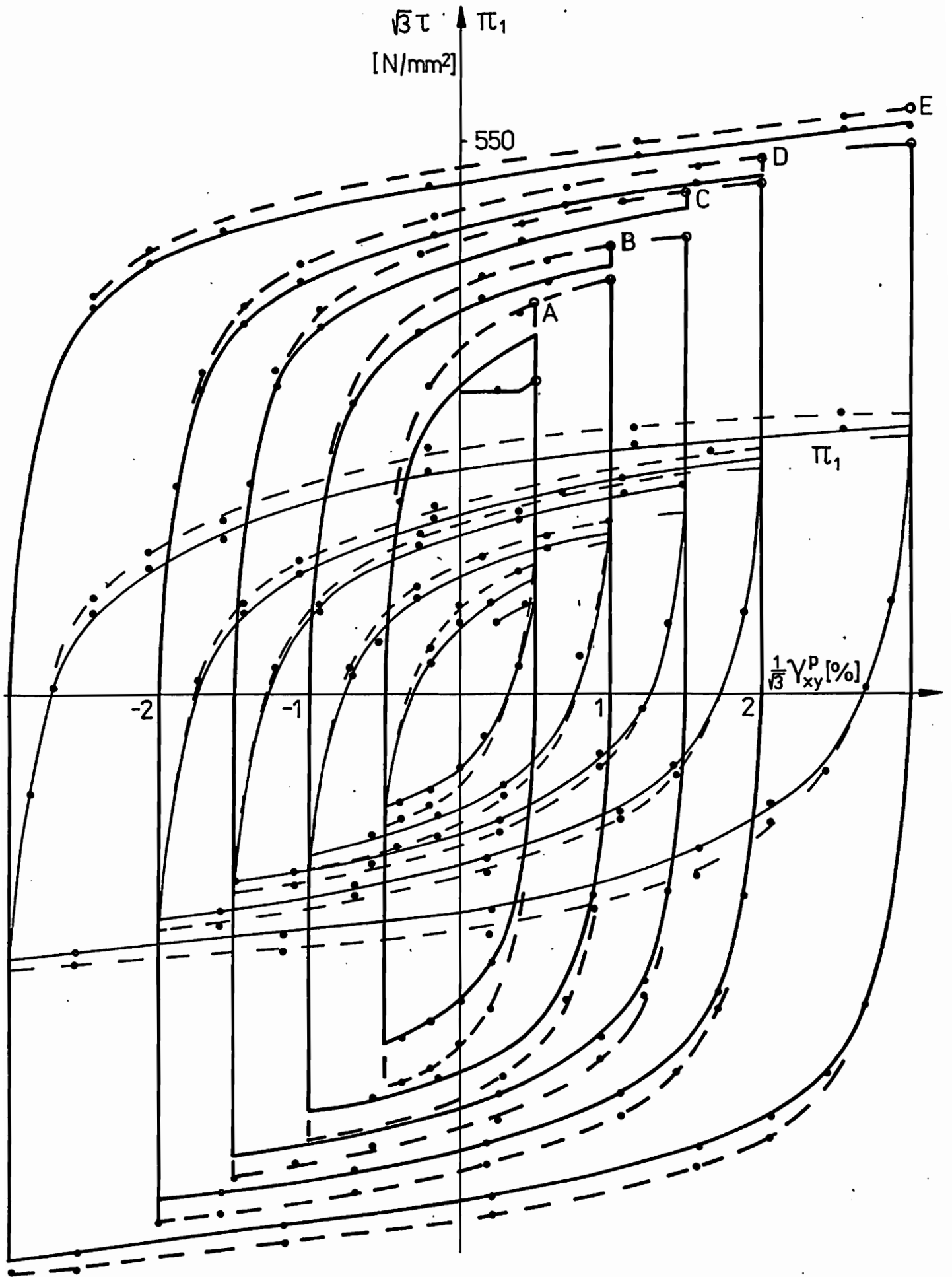
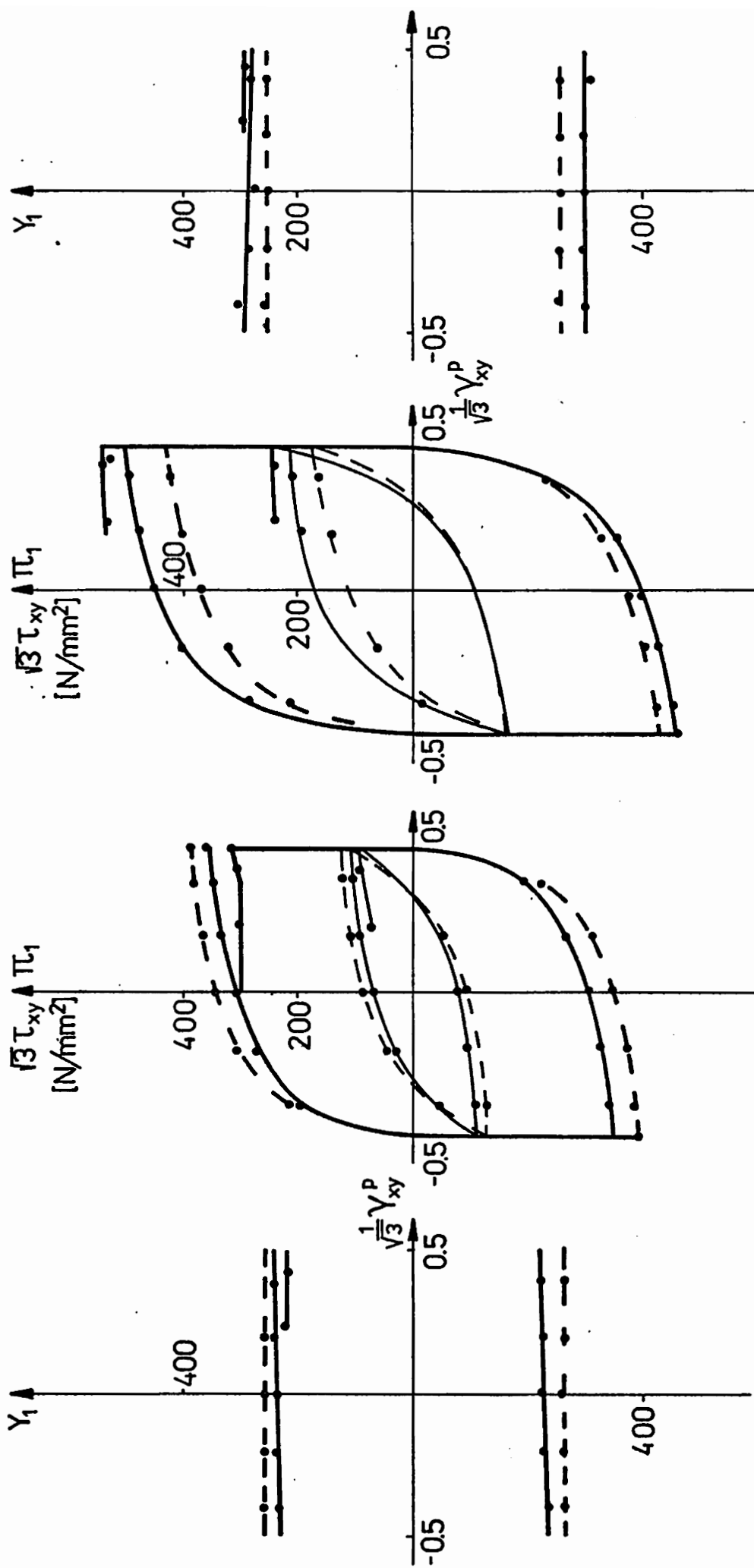


Fig.5



b)

a)

Fig.6

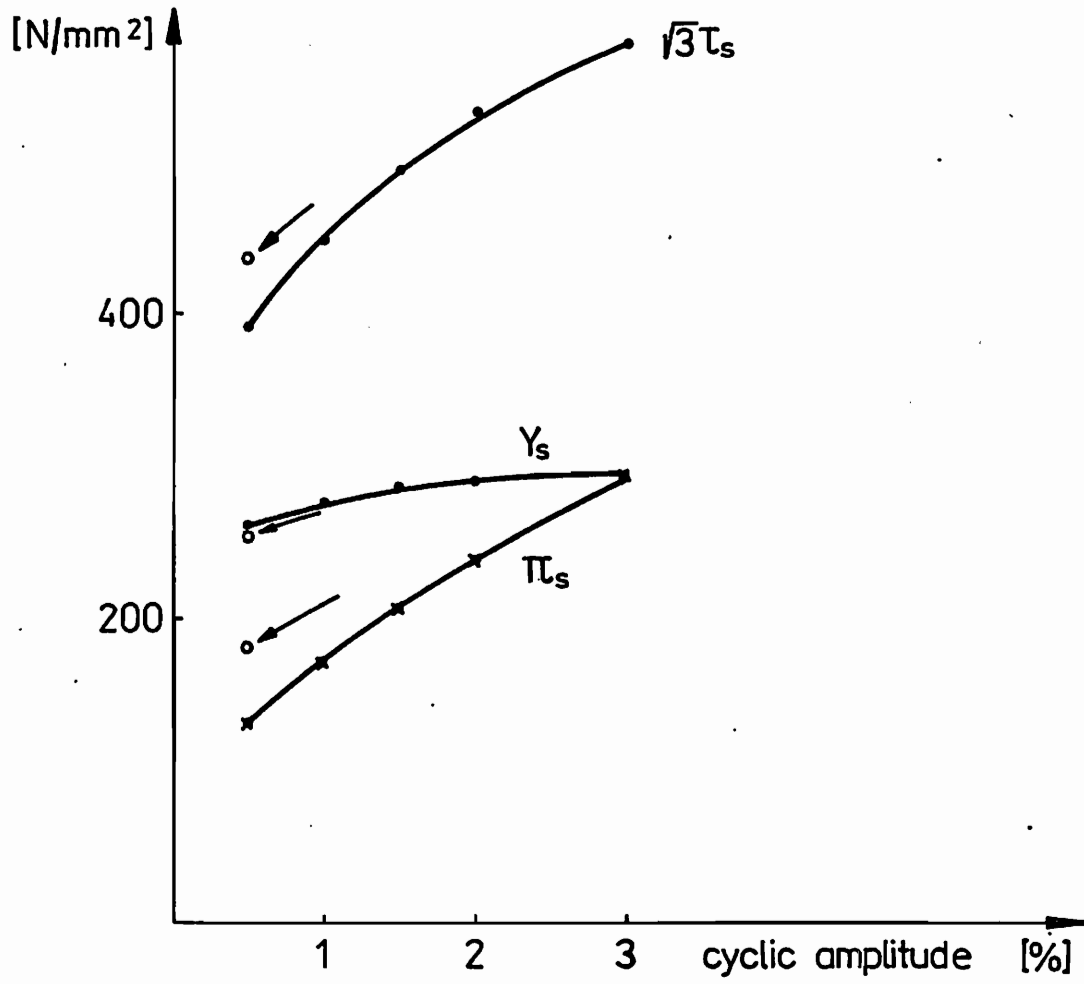


Fig.7

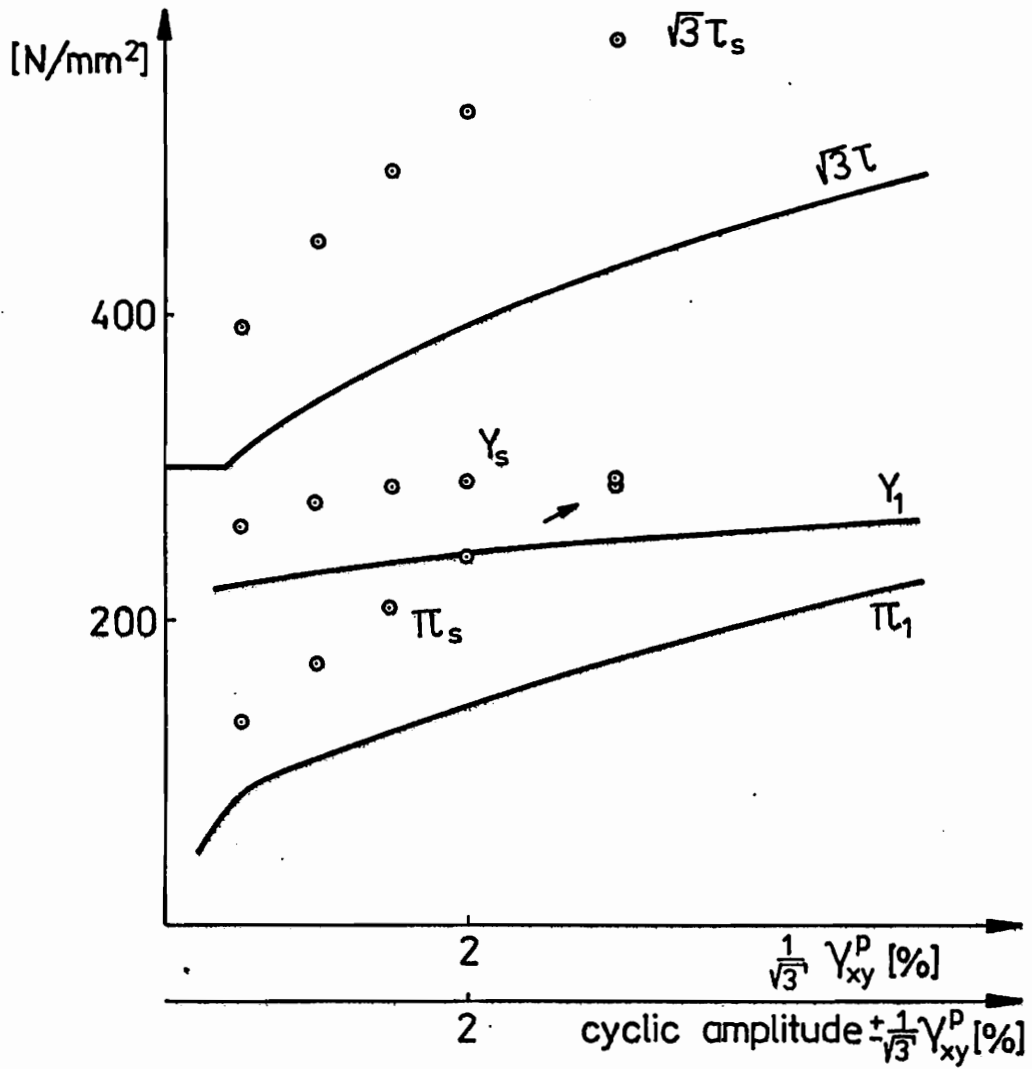


Fig. 8



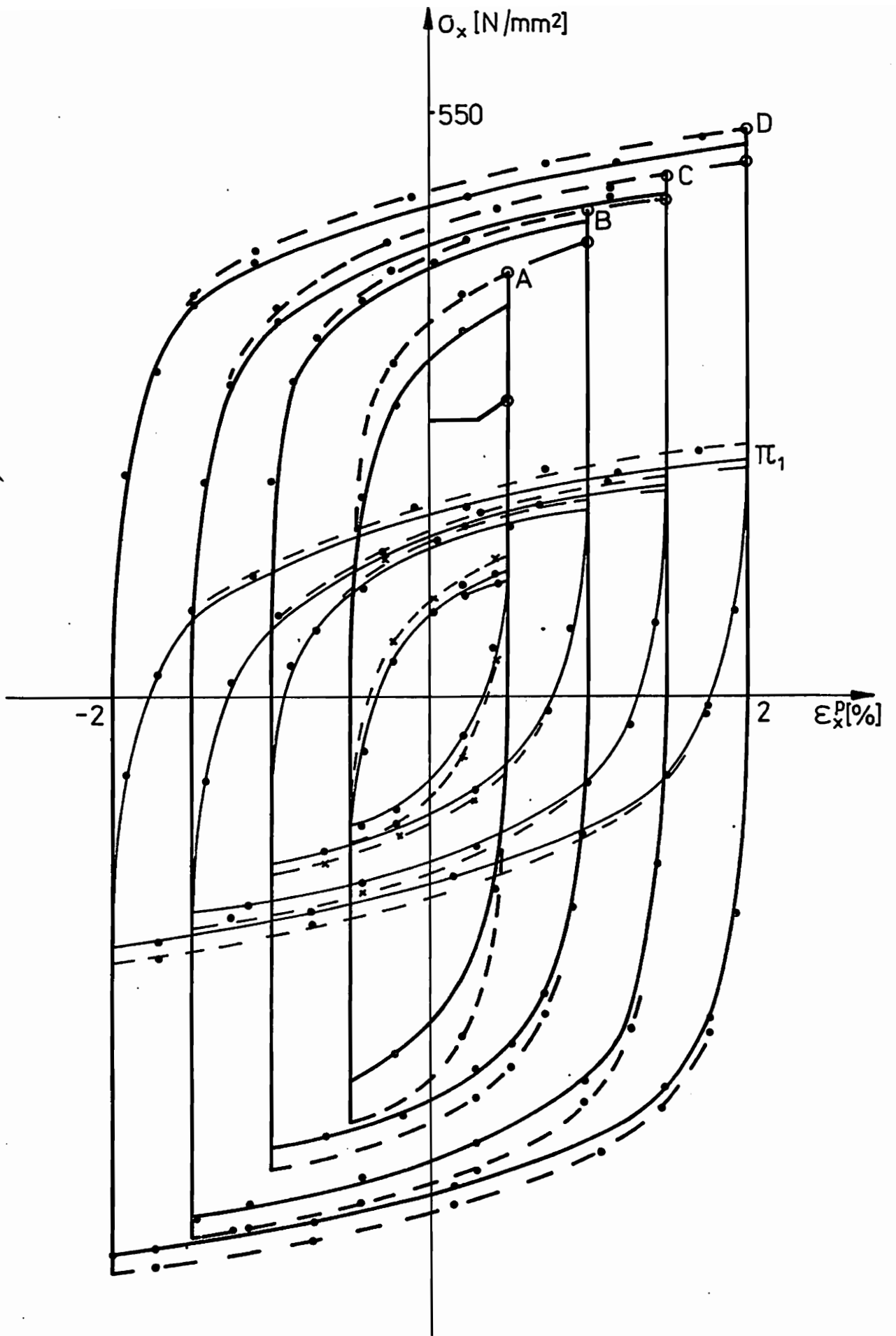


Fig.9

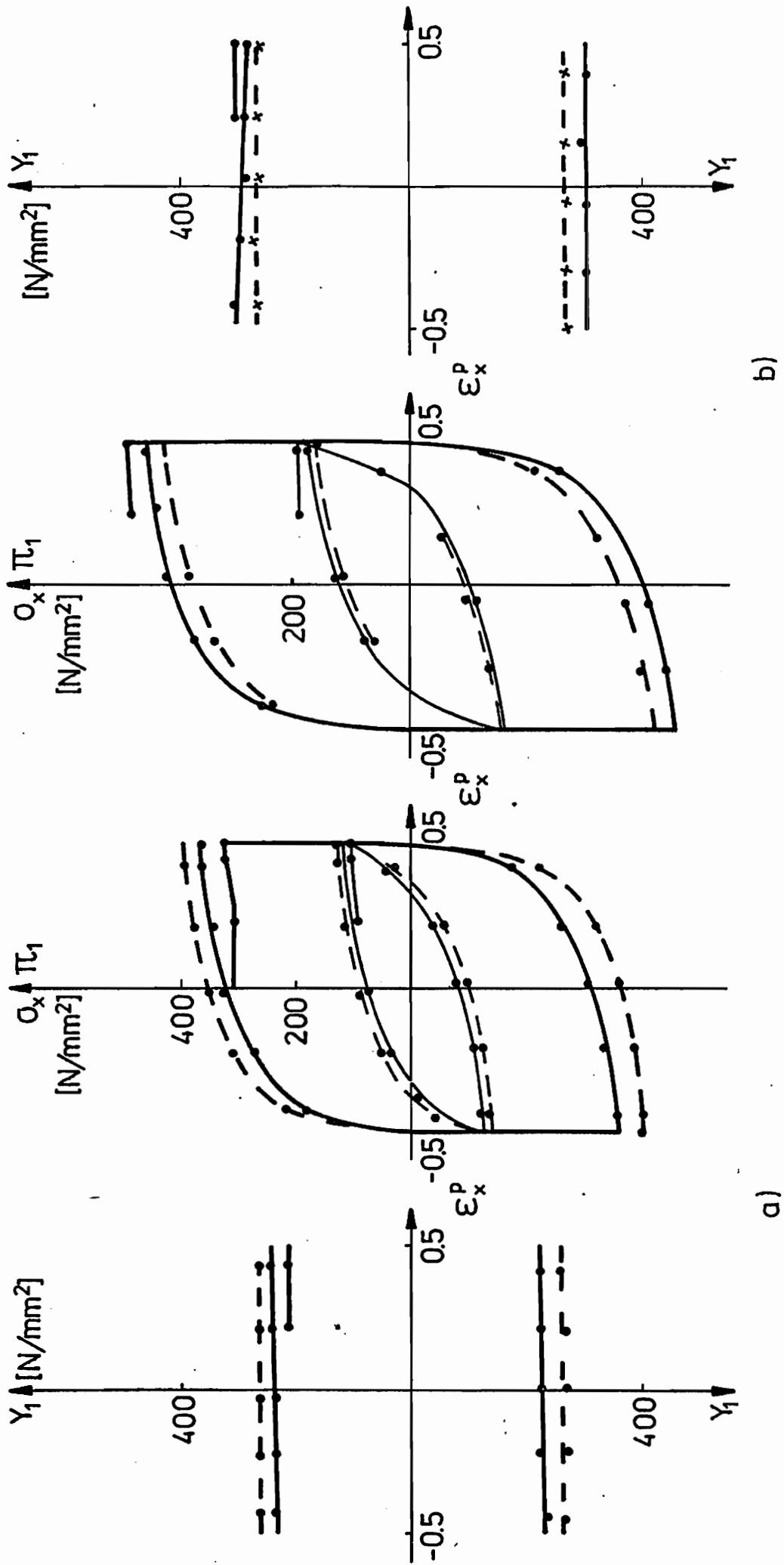


Fig.10

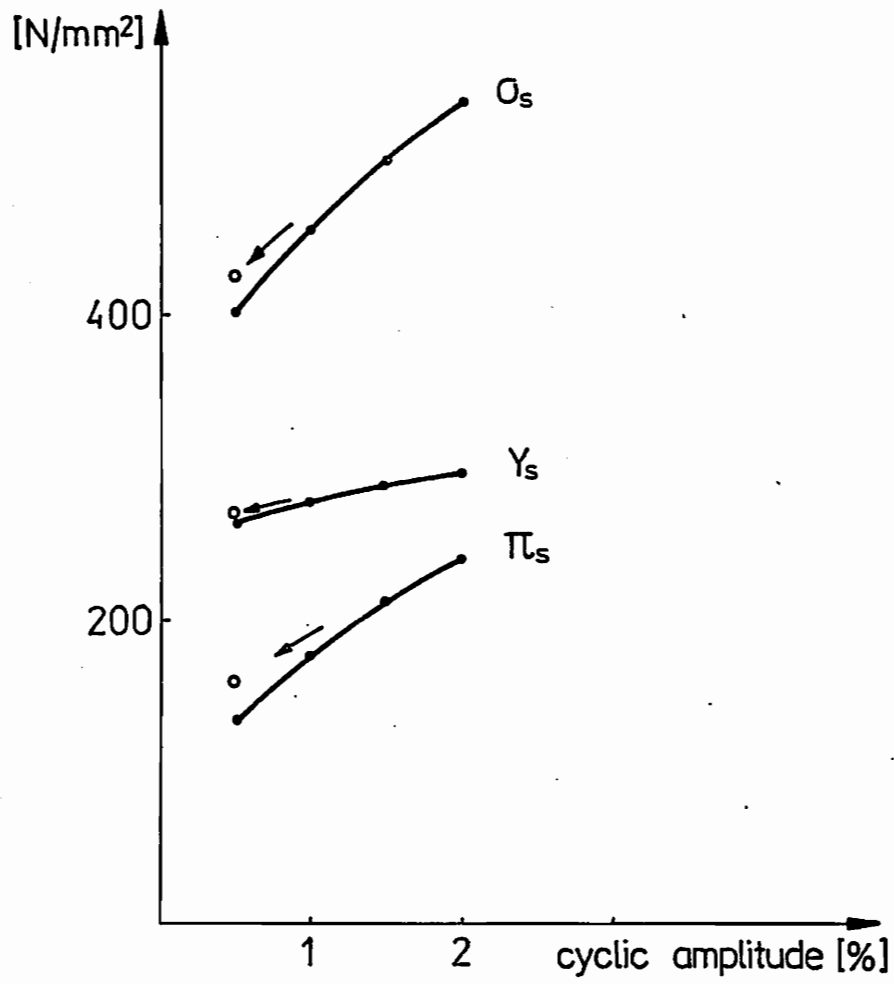


Fig.11

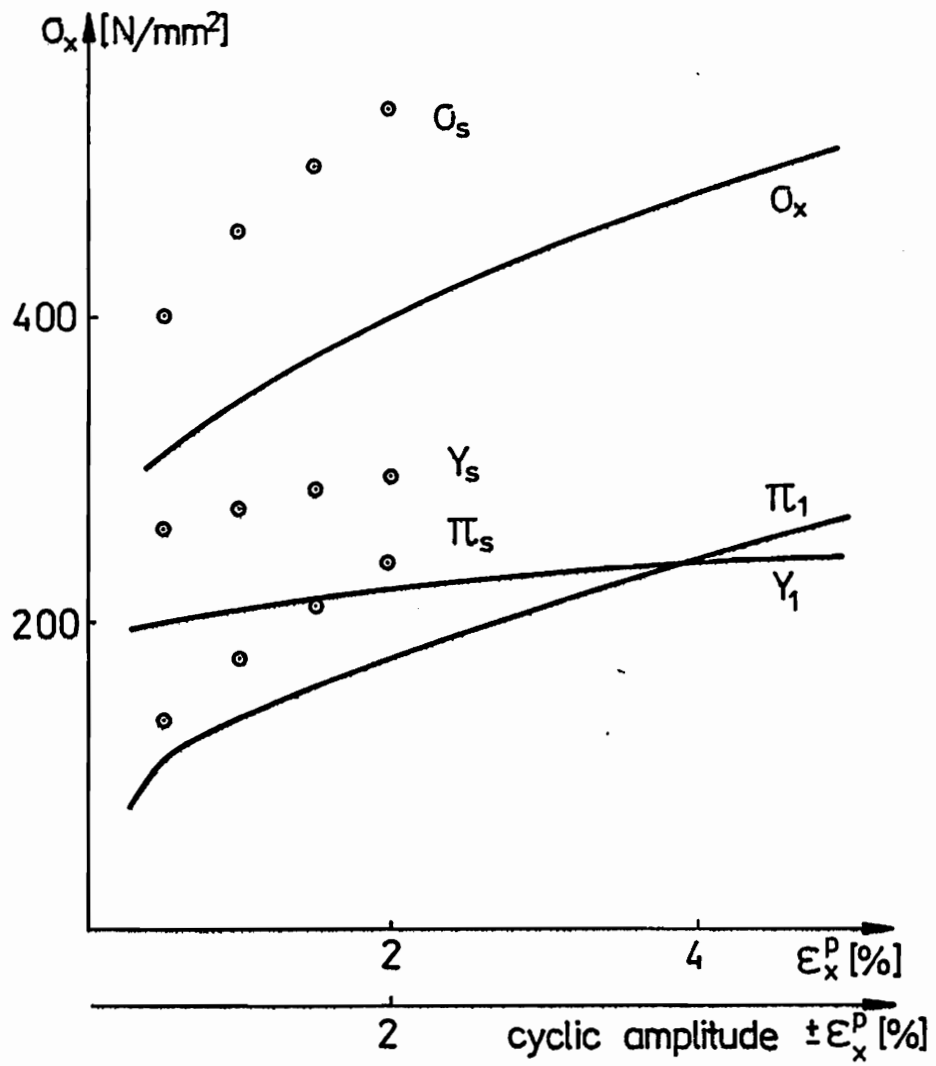


Fig.12

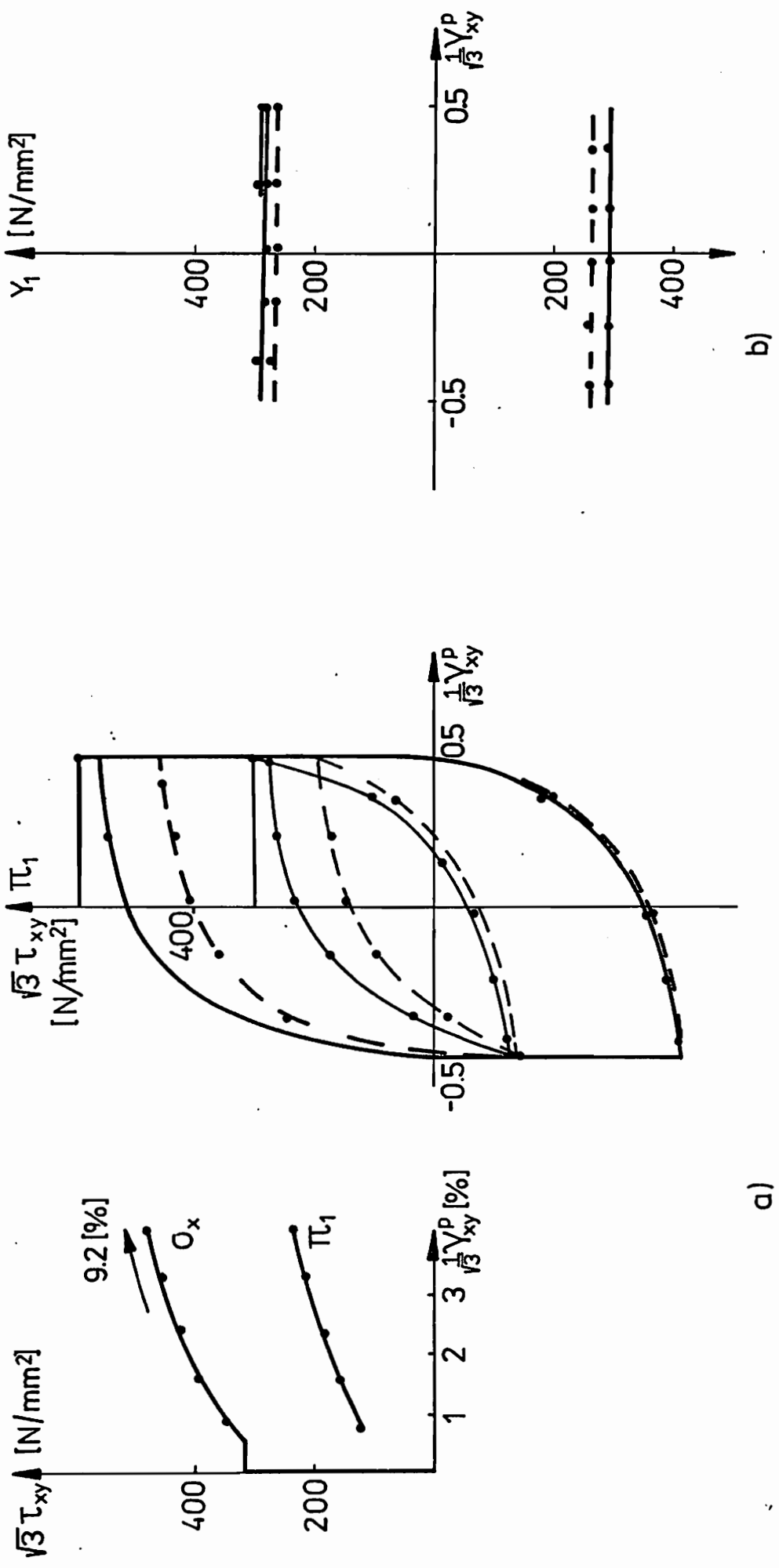
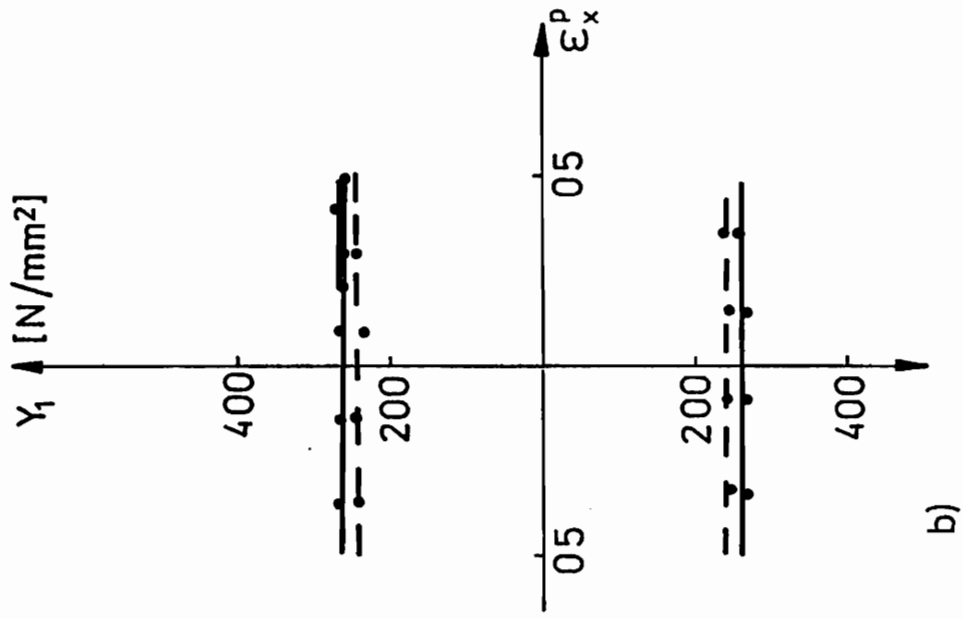
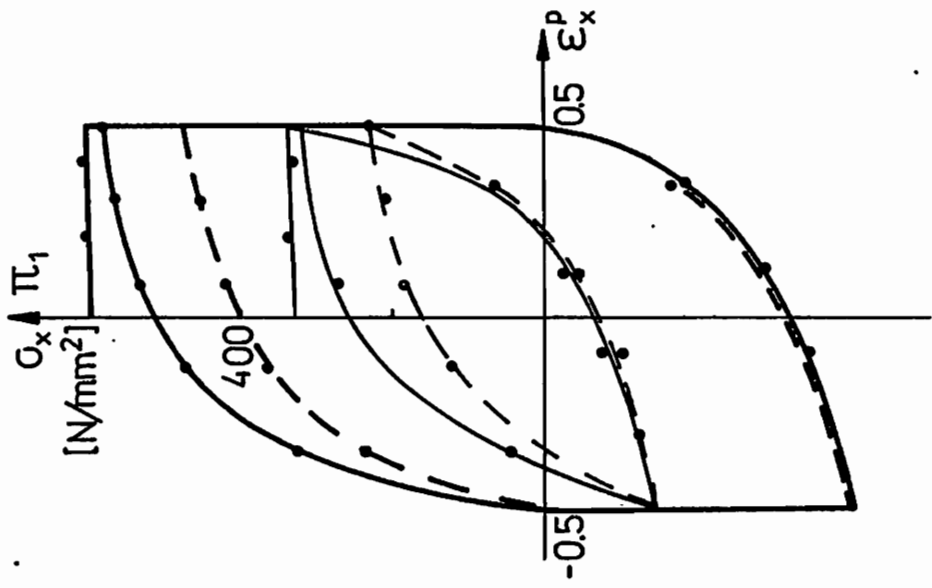
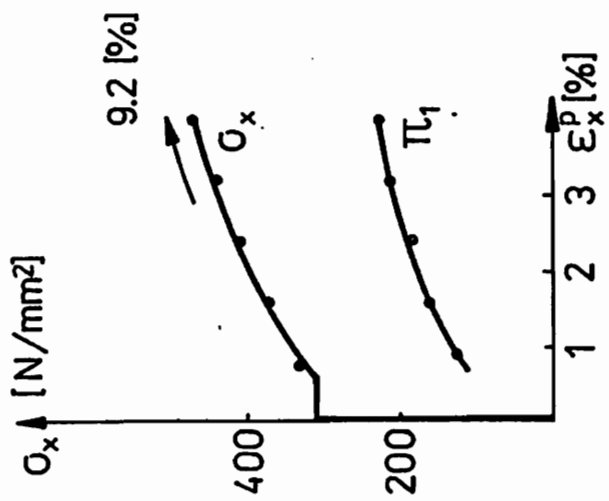


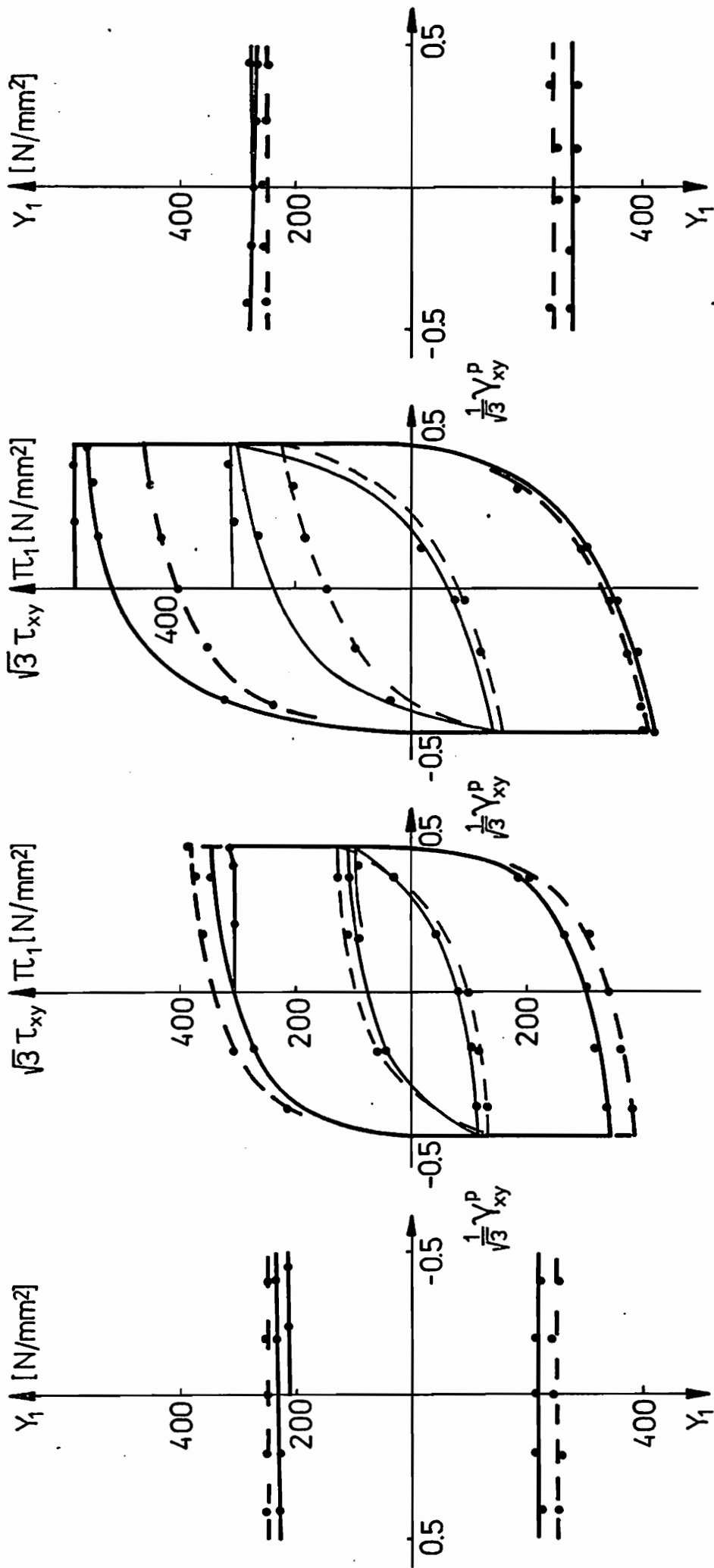
Fig. 13



a)

b)

Fig.14



b)

a)

Fig. 15

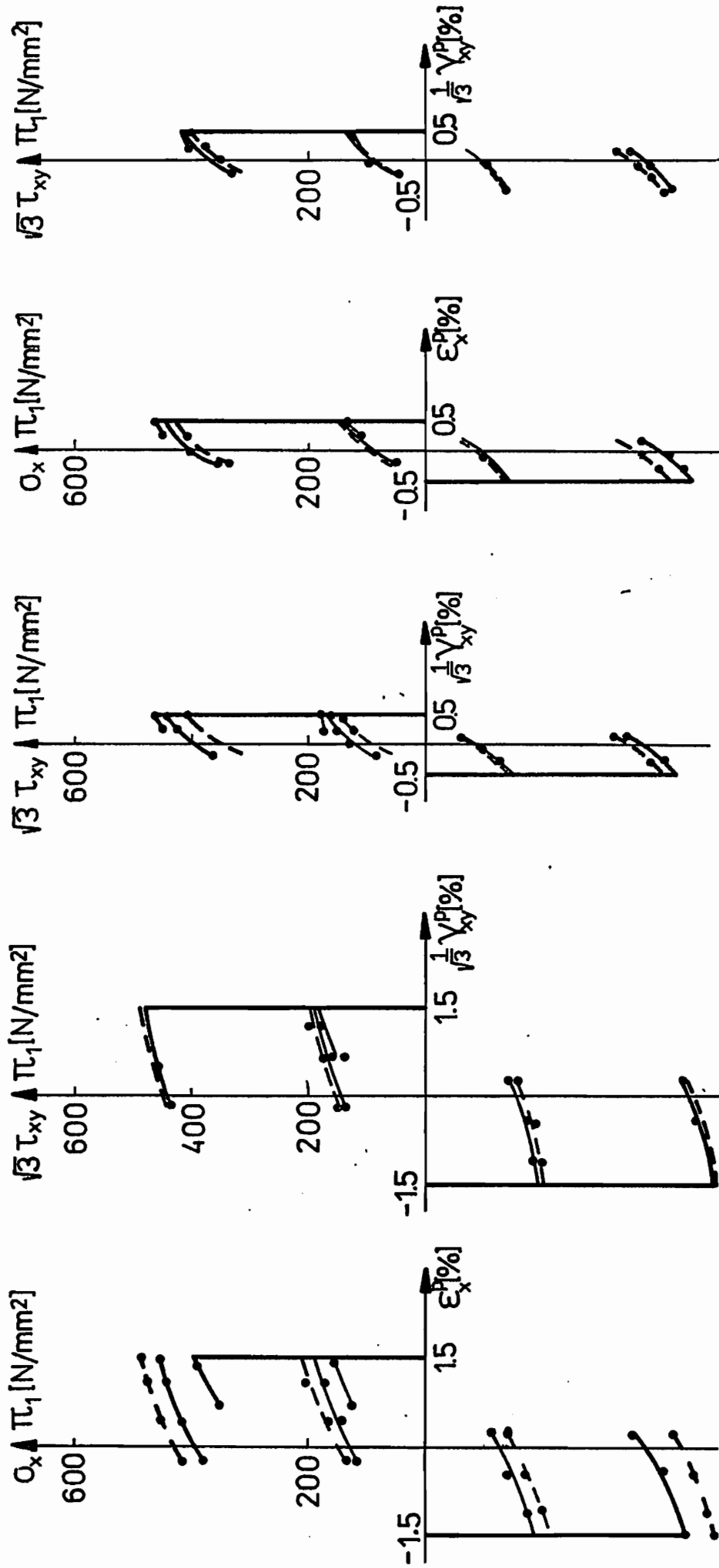


Fig. 16.a



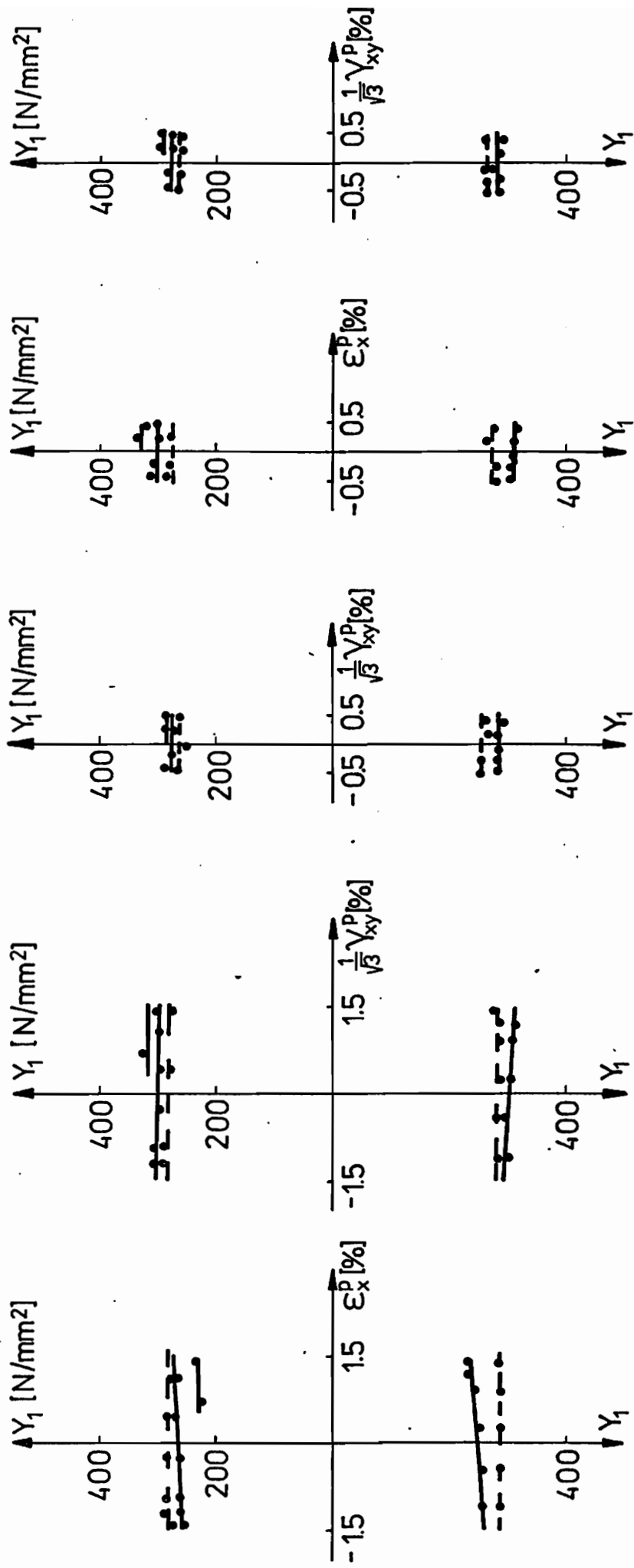


Fig.16.b

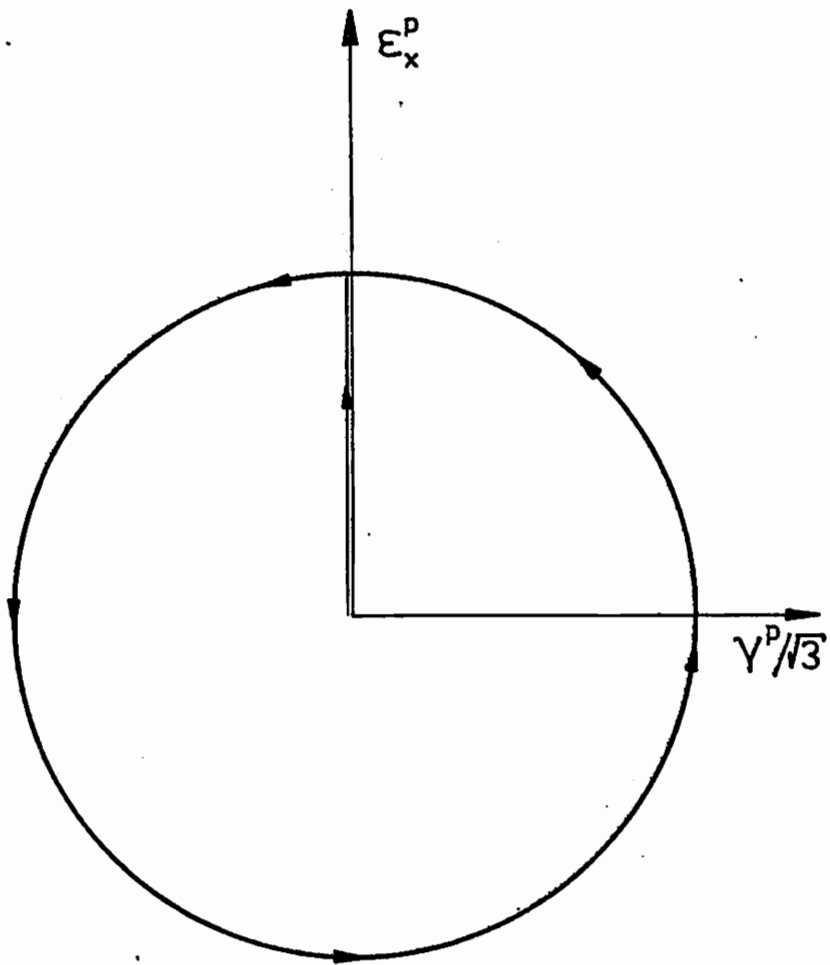


Fig. 17

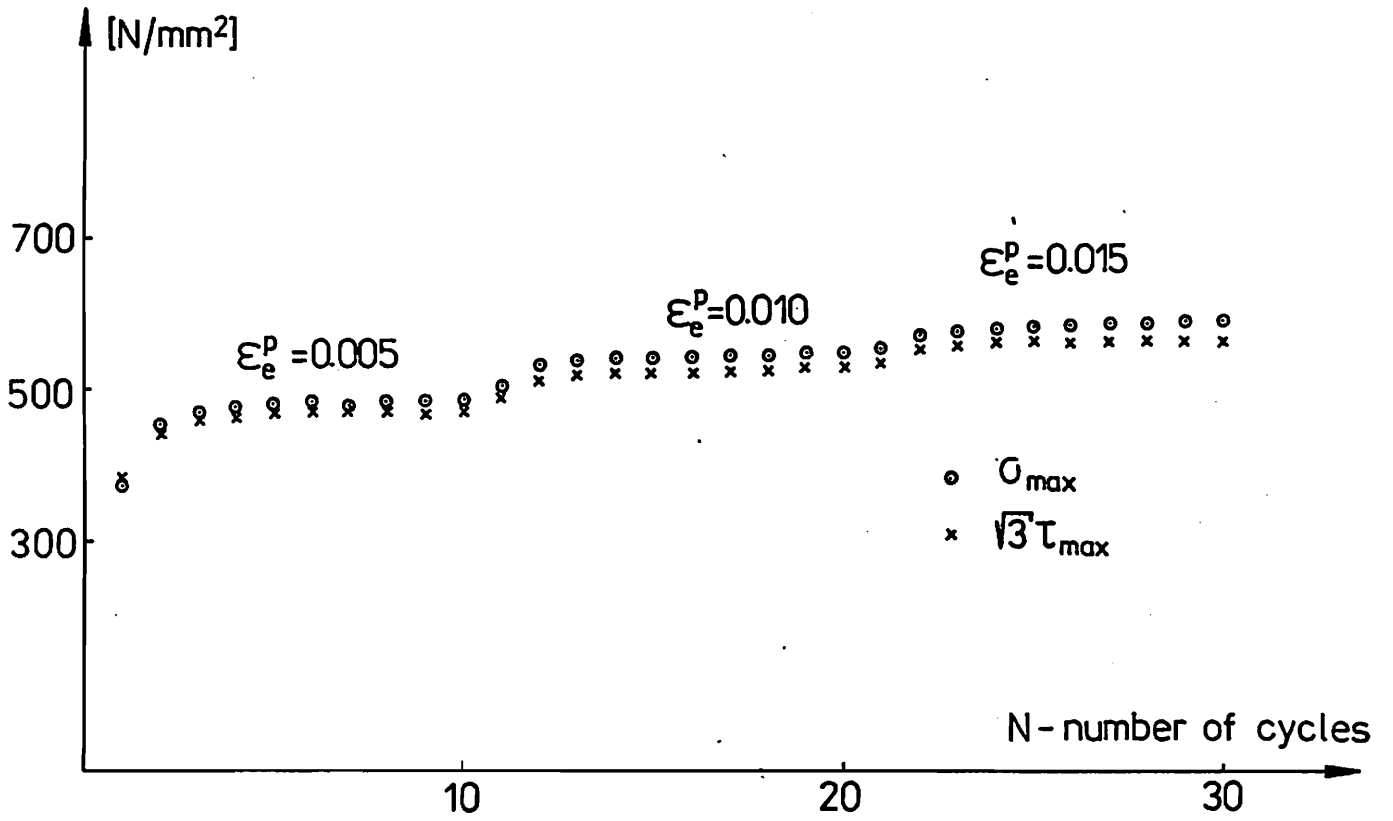


Fig. 18

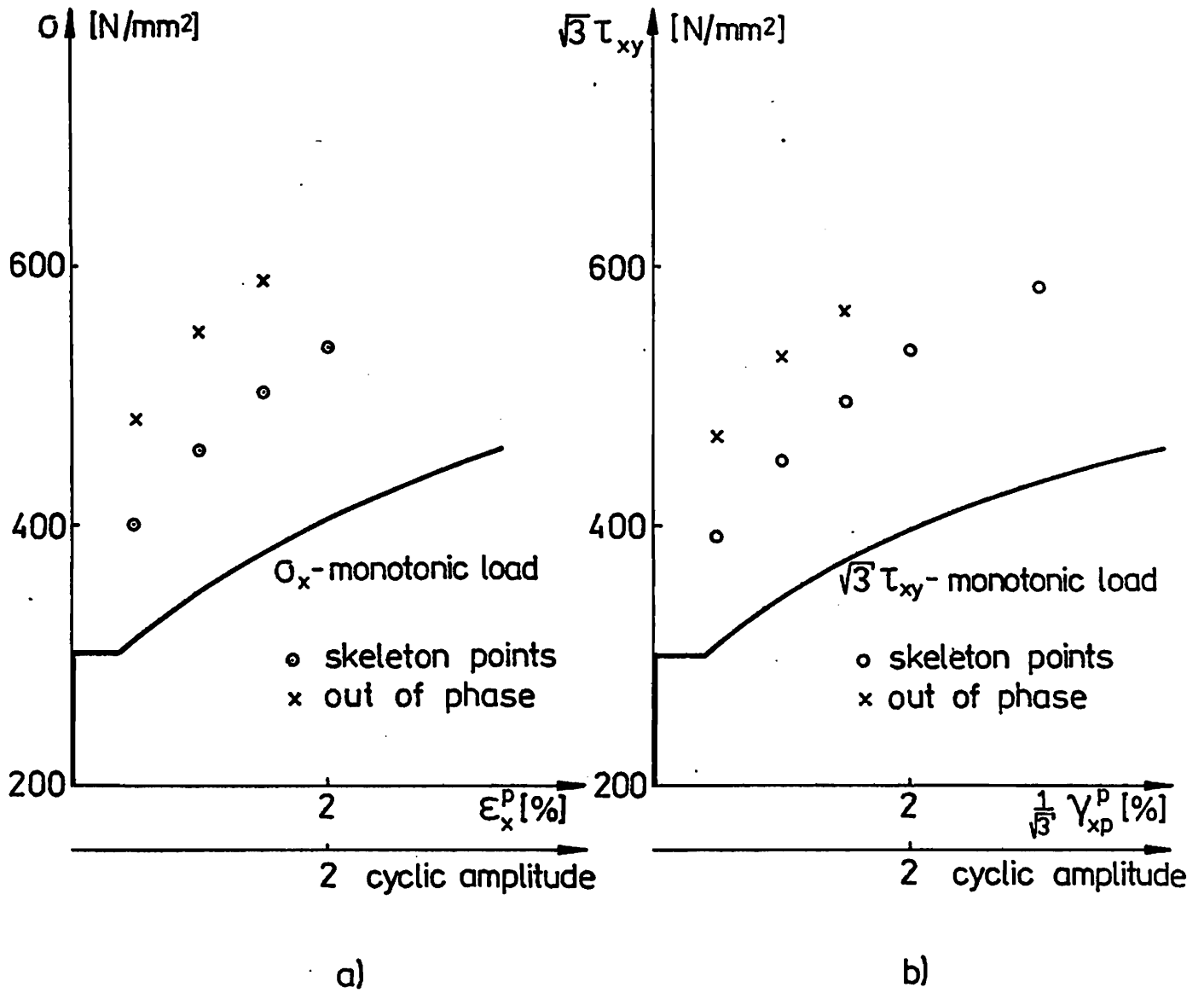


Fig. 19

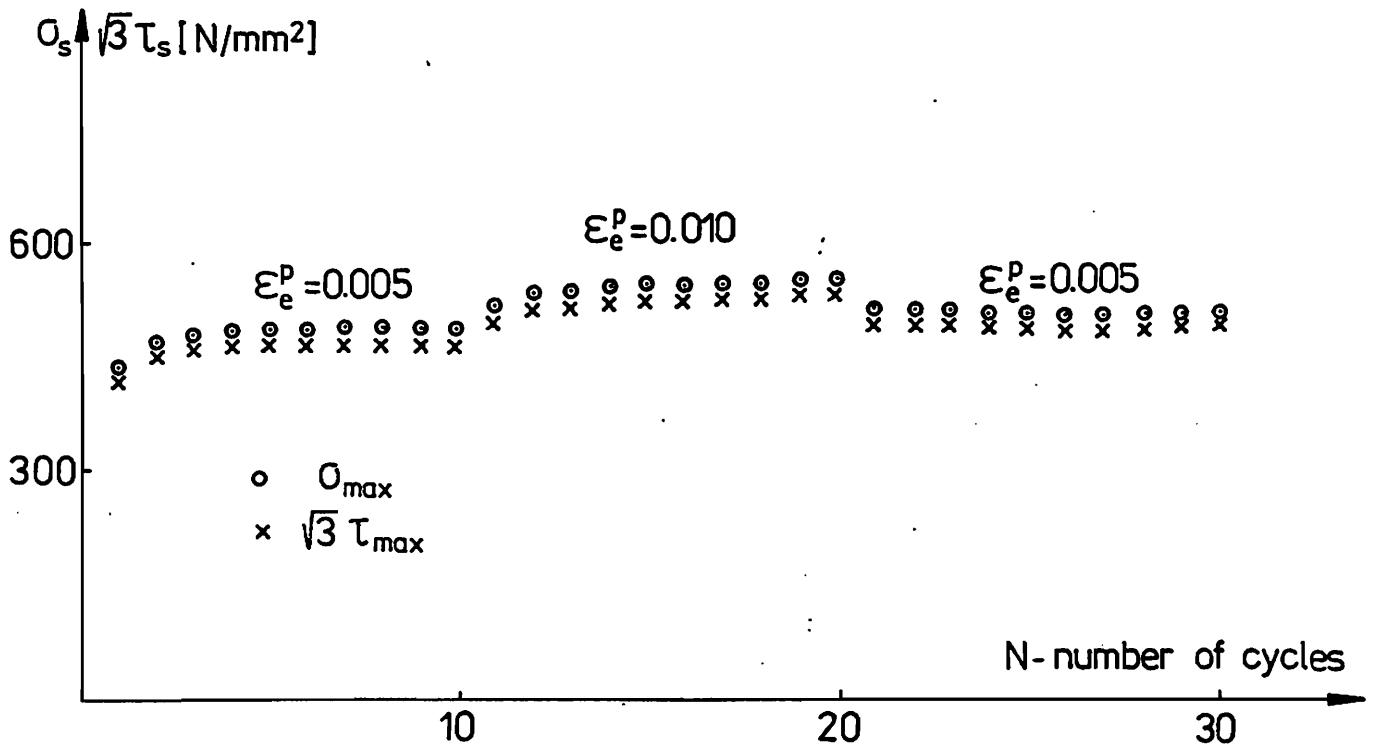


Fig.20

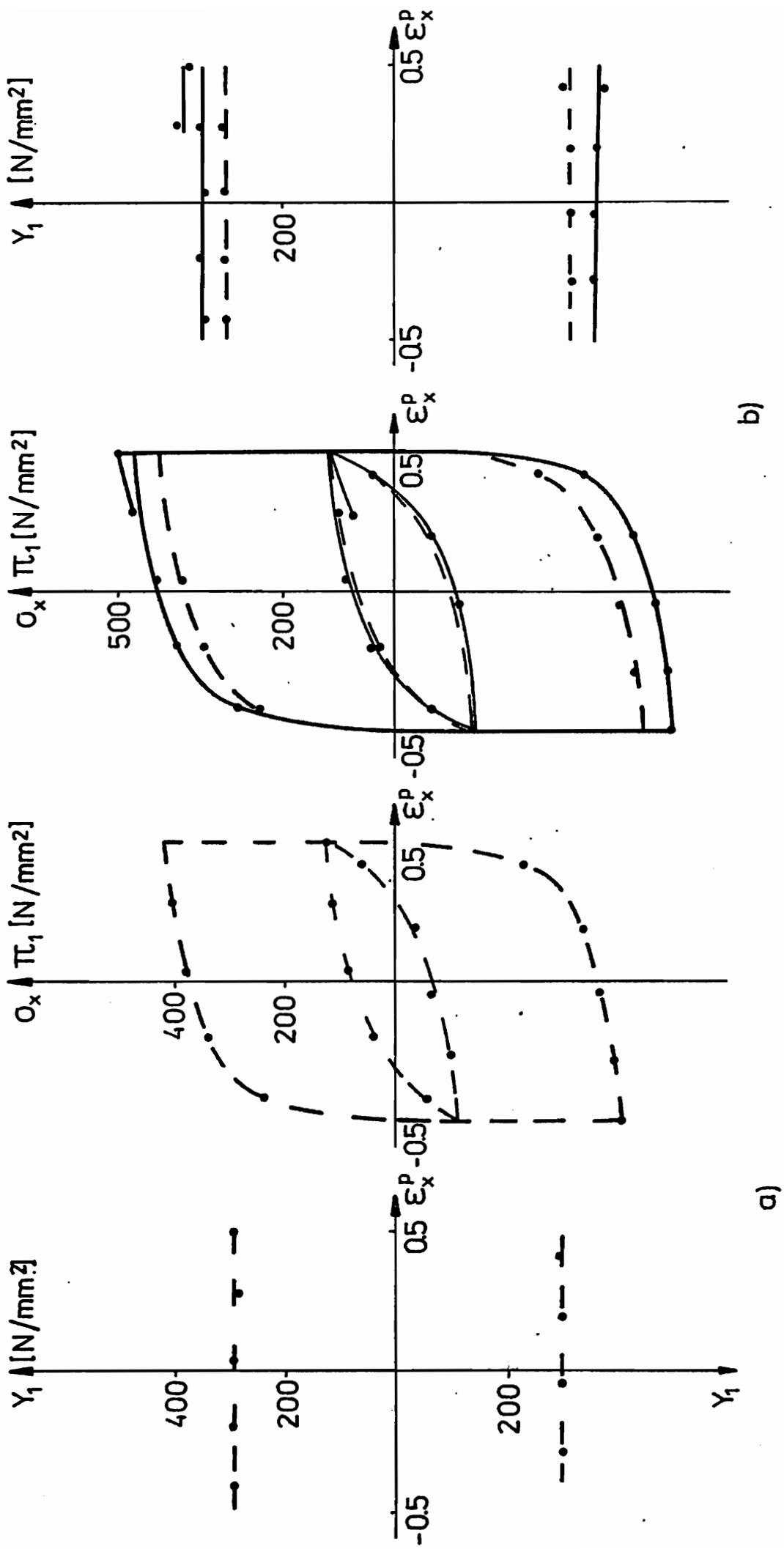


Fig. 21

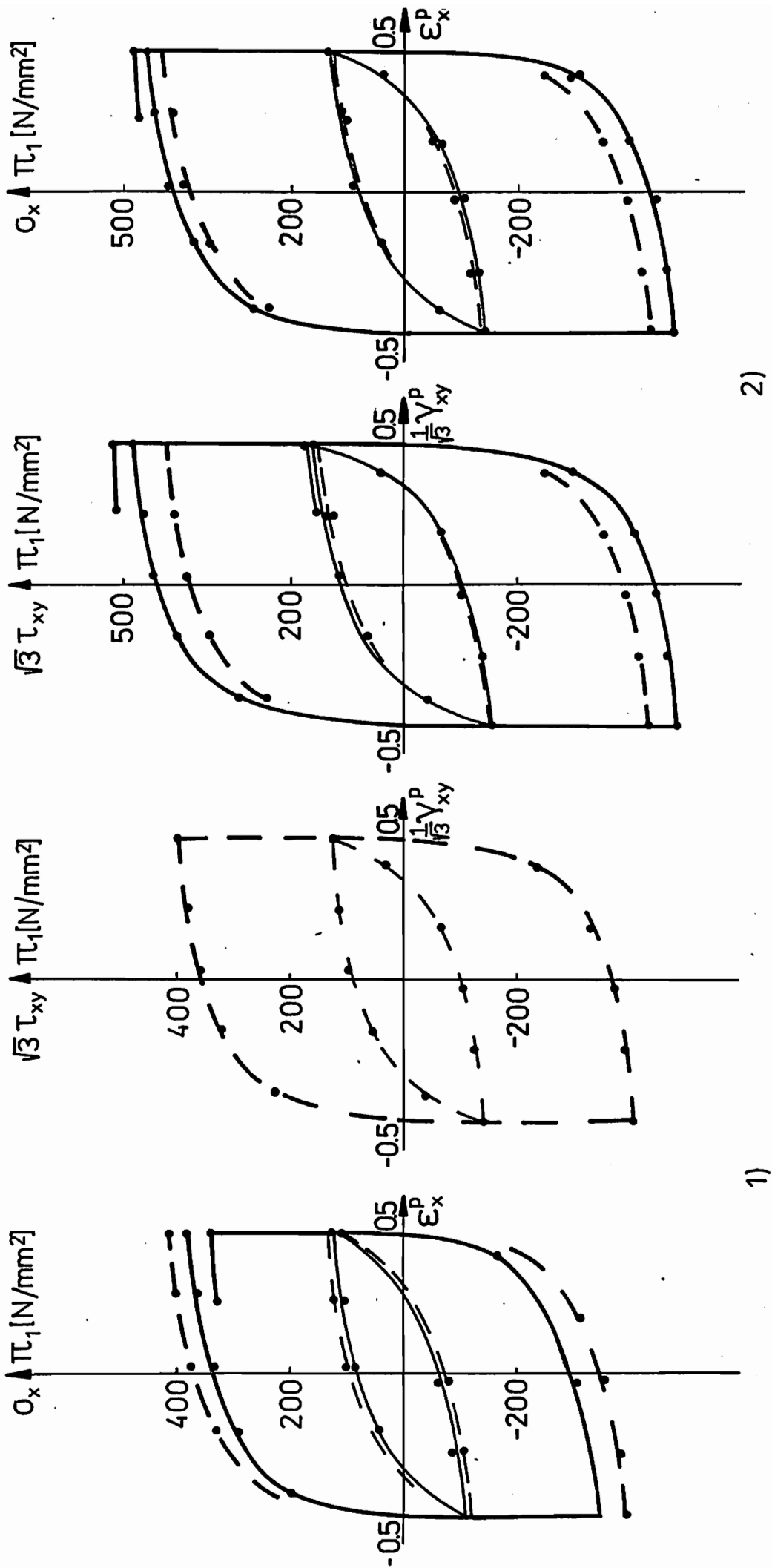


Fig. 22.a

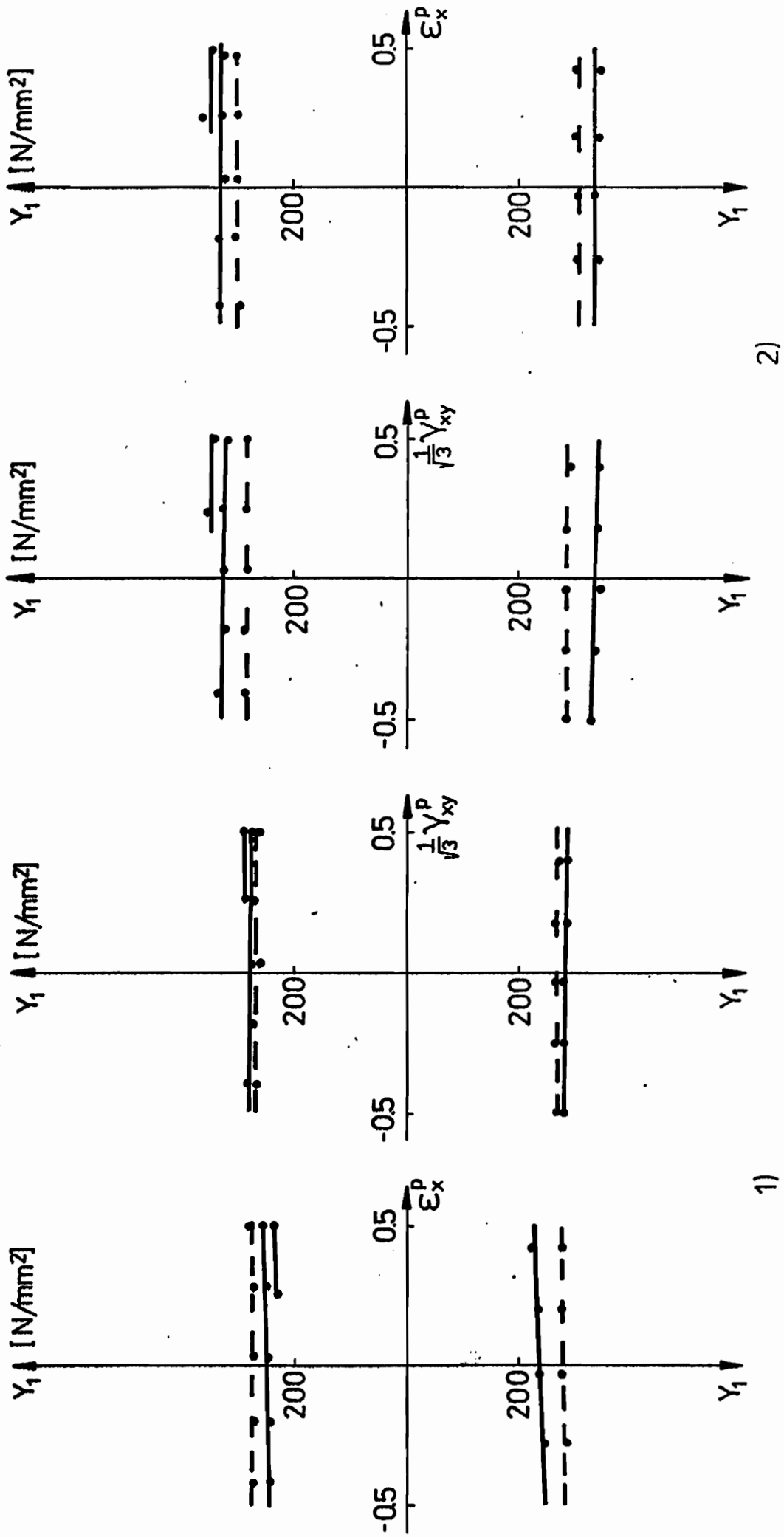


Fig.22.b



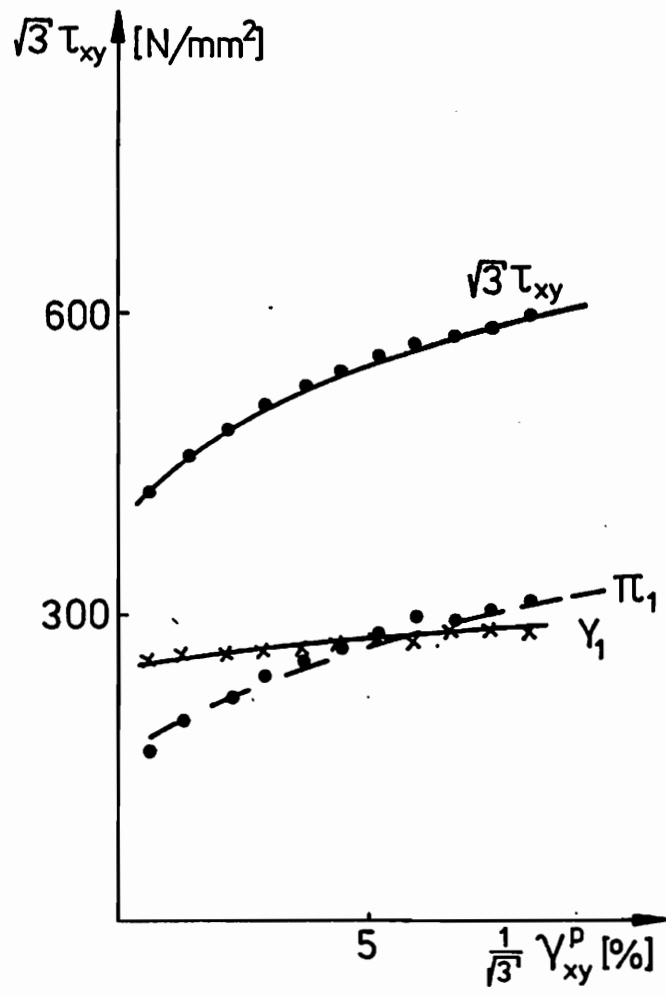


Fig.23

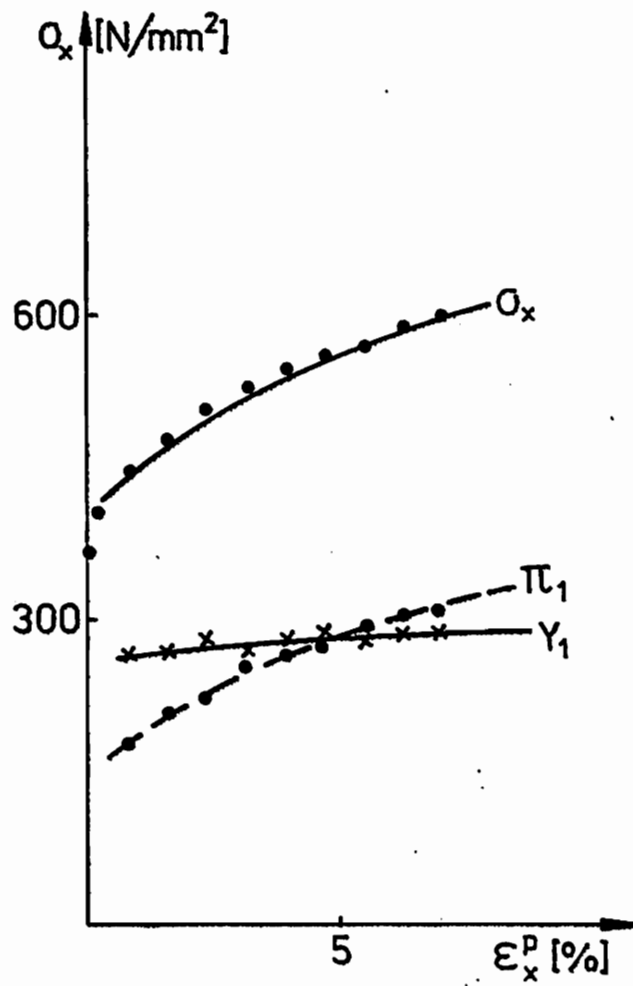


Fig.24

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